Necessary Conditions of Line Congestions in Uncertainty Accommodation

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***Abstract*—In this letter, a set of general necessary conditions for line congestions are established during ramping delivery, i.e., the redispatch process for uncertainty accommodation. Mathematically, the consideration of all line flow constraints may cause the intractability of the max-min problem in robust security-constrained unit commitment and dispatch. Without solving time-consuming linear programming (LP) problems, the lines that will not be congested can be quickly identified based on the necessary conditions and corresponding line flow constraints can be removed from the max-min subproblem in robust approaches. A promising application of the conditions is presented via the preprocessed max-min problem with high computational performance.**

***Index Terms*—Line congestion, ramping delivery, reserve, robust optimization.**

I. Introduction

Due to forecasting errors, the system operator must maintain certain reserves to accommodate the uncertainties caused by renewable energy sources (RES). Robust optimization approaches considering uncertainties are studied extensively to determine the optimal reserves. The robustness is obtained by solving a max-min problem, where the worst uncertainty point is found. Although the extreme point based method is adopted to linearize the nonconvex max-min problem [1], the converted mixed-integer linear programming (MILP) problem is still intractable [2], [3]. Recognizing the challenge is from line flow constraints and inspired by [4], [5], we propose a set of necessary conditions for identifying line congestions in this letter. The lines that will not be congested for uncertainty accommodation can be quickly identified based on the necessary conditions instead of relying on experiences or heuristics. By eliminating the flow constraints of these non-congested lines, the robust approach is effectively accelerated.

II. Congestion Identification

Consider that the economic dispatch (ED) P_i for unit i is given and it can be adjusted by ΔP_i to accommodate uncertainties [3]. ΔP_i can also be explained as the available reserves. Let ϵ_m and d_m denote the uncertainty and load at bus m, respectively. Consider the uncertainty set $\mathcal{U} := \mathcal{U}^{\mathrm{b}} \cap \mathcal{U}^{\mathrm{c}}$, where $\mathcal{U}^{\mathrm{b}} := \{\boldsymbol{\epsilon} \in \mathbb{R}^{N_d} : \underline{u}_m \le \epsilon_m \le \bar{u}_m\}$ and $\mathcal{U}^{\mathrm{c}}$ includes other constraints on uncertainties. N_d is the number of buses and $\underline{u}_m \le 0 \le \bar{u}_m$. A max-min problem (W) to find the worst uncertainty point is formulated

$$(W): \quad \max_{\boldsymbol{\epsilon} \in \mathcal{U}} \quad \min_{(\boldsymbol{\Delta P}, \boldsymbol{s}^+, \boldsymbol{s}^-) \in \mathcal{F}(\boldsymbol{P}, \boldsymbol{\epsilon})} \sum_m \left(s_m^+ + s_m^-\right), \tag{1}$$

where

$$\mathcal{F}(\boldsymbol{P}, \boldsymbol{\epsilon}) := \big\{(\boldsymbol{\Delta P}, \boldsymbol{s}^+, \boldsymbol{s}^-) : \boldsymbol{\epsilon} + \boldsymbol{s}^+ - \boldsymbol{s}^- \in \mathcal{U}, s_m^+, s_m^- \ge 0$$

$$\sum_i (P_i + \Delta P_i) = \sum_m \left(d_m + \epsilon_m + s_m^+ - s_m^-\right) \tag{2}$$

$$P_m^{\mathrm{inj}} = \sum_{i \in \mathcal{G}(m)} (P_i + \Delta P_i) - d_m - \epsilon_m - s_m^+ + s_m^-, \ \forall m \tag{3}$$

$$-F_l \le \sum_m \left(\Gamma_{l,m} P_m^{\mathrm{inj}}\right) \le F_l, \ \forall l \tag{4}$$

$$\underline{r}_i \le \Delta P_i \le \bar{r}_i, \ \forall i\big\} \tag{5}$$

where s_m^+ and s_m^- are un-accommodated uncertainties (or generation and load curtailments). (2) denotes the load balance constraint. The net power injection P_m^{inj} in (3) is subject to the line flow constraint (4), where $\Gamma_{l,m}$ and F_l are the shift factor and line capacity, respectively. $\mathcal{G}(m)$ is the set of units located at bus m. ΔP_i is limited by (5), where $\underline{r}_i$ and $\bar{r}_i$ are functions of the ramping limits and generation capacities. The non-convex (W) is NP-hard and generally difficult to solve.

If (4) is dropped, then the solution to (W) is trivial. The worst uncertainty point is reached when $\sum_m \epsilon_m$ is at the upper/lower bound. However, if (4) is enforced, the reserves may not be delivered to some buses due to line congestions. Different from the single-level problem in [4], [5], the line congestion here occurs in a two-level max-min problem (W). The outer-level maximizes the curtailment by selecting $\boldsymbol{\epsilon} \in \mathcal{U}$. The inner-level minimizes the curtailment by determining ΔP_i. The congestion in the positive direction is considered in the following context, and the same analysis applies to the congestion in the negative direction.

Theorem 1: If line l is congested in the positive direction for uncertainty accommodation, the optimal value z_1 to problem (P1) with relaxed uncertainty set $\mathcal{U}^{\mathrm{b}}$ must be non-negative.

$$(P1): \quad z_1 := \max_{\boldsymbol{\Delta P}, \boldsymbol{\epsilon} \in \mathcal{U}^{\mathrm{b}}} \quad \sum_m \left(\Gamma_{l,m} P_m^{\mathrm{inj}}\right) - F_l \tag{6}$$

$$\text{s.t.} \quad \sum_i (P_i + \Delta P_i) = \sum_m (d_m + \epsilon_m) \tag{7}$$

$$P_m^{\mathrm{inj}} = \sum_{i \in \mathcal{G}(m)} (P_i + \Delta P_i) - d_m - \epsilon_m, \ \forall m \tag{8}$$

$$\underline{r}_i \le \Delta P_i \le \bar{r}_i, \ \forall i \tag{9}$$

Proof: Assume the optimal point to problem (W) is $\tilde{\boldsymbol{\epsilon}}$. The optimal point $(\tilde{\boldsymbol{s}}^+(\tilde{\boldsymbol{\epsilon}}), \tilde{\boldsymbol{s}}^-(\tilde{\boldsymbol{\epsilon}}), \boldsymbol{\Delta}\tilde{\boldsymbol{P}}(\tilde{\boldsymbol{\epsilon}}), \tilde{\boldsymbol{P}}^{\mathrm{inj}}(\tilde{\boldsymbol{\epsilon}}))$ to the inner problem in (W) is a function of $\tilde{\boldsymbol{\epsilon}}$. Denote the feasible set of (P1) as $\mathcal{F}_1$. It can be verified $(\boldsymbol{\Delta}\tilde{\boldsymbol{P}}(\tilde{\boldsymbol{\epsilon}}), \tilde{\boldsymbol{\epsilon}} + \tilde{\boldsymbol{s}}^+(\tilde{\boldsymbol{\epsilon}}) - \tilde{\boldsymbol{s}}^-(\tilde{\boldsymbol{\epsilon}})) \in \mathcal{F}_1$. Therefore, $z_1 \ge \sum_m \Gamma_{l,m} \tilde{P}_m^{\mathrm{inj}}(\tilde{\boldsymbol{\epsilon}}) - F_l$. So, if line l is congested in (W), (i.e.$\sum_m \Gamma_{l,m} \tilde{P}_m^{\mathrm{inj}}(\tilde{\boldsymbol{\epsilon}}) - F_l = 0$), then $z_1 \ge 0$ holds. □

If $\mathcal{U}^b$ in (P1) is replaced with $\mathcal{U}$, the above proof is still valid and the condition is stronger but with heavier computation burden. By adding other line constraints in (P1), we can further get a sufficient and necessary condition for the redundancy of the network constraint.

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Theorem 1 is not practically useful as it is time consuming to solve a large number of (P1). Next, a more efficient condition equivalent to Theorem 1 will be established. By introducing new variables $\hat{\epsilon}_m := \epsilon_m - \bar{u}_m$ and $\hat{P}_i := \Delta P_i - \underline{r}_i$, the net power injection can be expressed as

$$\begin{cases} P_m^{\text{inj}} = \hat{P}_m^{\text{inj}} - \hat{D}_m, \quad \hat{P}_m^{\text{inj}} = \sum_{i\in\mathcal{G}(m)} \hat{P}_i - \hat{\epsilon}_m, \forall m \\ \hat{D}_m = \bar{u}_m + d_m - \sum_{i\in\mathcal{G}(m)} (\underline{r}_i + P_i), \forall m \end{cases}$$

Let $\bar{P}_m^{\text{inj}}$ denote the upper bound of $\hat{P}_m^{\text{inj}}$. As $\underline{u}_m - \bar{u}_m \le \hat{\epsilon}_m \le 0$ and $0 \le \hat{P}_i \le \bar{r}_i - \underline{r}_i$, then

$$0 \le \hat{P}_m^{\text{inj}} \le \bar{P}_m^{\text{inj}} = \sum_{i\in\mathcal{G}(m)} (\bar{r}_i - \underline{r}_i) - \underline{u}_m + \bar{u}_m, \forall m \quad (10)$$

holds. Denote $D := \sum_m \hat{D}_m$ and $f_l := \sum_m \Gamma_{l,m} \hat{D}_m$, then problem (P1) can be rewritten as a new problem (P2)

$$(P2): \quad z_2 := \max_{\hat{P}_m^{\text{inj}}, \forall m} \quad -F_l - f_l + \sum_m \Gamma_{l,m} \hat{P}_m^{\text{inj}}$$
$$\text{s.t.} \quad 0 \le \hat{P}_m^{\text{inj}} \le \bar{P}_m^{\text{inj}}, \forall m, \quad \sum_m \hat{P}_m^{\text{inj}} = D.$$

(P2) is equivalent to (P1), and $z_1 = z_2$. Once $\Gamma_{l,m}$ is ordered, (P2) can be solved with complexity $\mathcal{O}(N_d)$ as shown below.

Theorem 2: If line l is congested in the positive direction for uncertainty accommodation, then the following conditions must hold for an integer $k \in [1, N_d]$.

$$\begin{cases} \sum_{n=1}^{k-1} \bar{P}_{m_n}^{\text{inj}} \le D \le \sum_{n=1}^{k} \bar{P}_{m_n}^{\text{inj}} & (11) \\ \sum_{n=1}^{k-1} (\Gamma_{l,m_n} - \Gamma_{l,m_k}) \bar{P}_{m_n}^{\text{inj}} + \Gamma_{l,m_k} D - F_l - f_l \ge 0 & (12) \\ \Gamma_{l,m_1} \ge \Gamma_{l,m_2} \ge \cdots \ge \Gamma_{l,m_{N_d}} & (13) \end{cases}$$

m_n is the bus with the nth largest shift factor for line l.

Proof: $(\mathbf{0}, \mathbf{0})$ is a feasible point in (P1), then (P2) is also feasible, which means that $0 \le D \le \sum_m \bar{P}_m^{\text{inj}}$ holds and k satisfying (11) must exist. Consider a k that satisfies (11). z_2 is obtained when $\hat{P}_{m_n}^{\text{inj}} = \bar{P}_{m_n}^{\text{inj}}, n \le k-1$; $\hat{P}_{m_k}^{\text{inj}} = D - \sum_{n=1}^{k-1} \bar{P}_{m_n}^{\text{inj}}$; $\hat{P}_{m_n}^{\text{inj}} = 0, n > k$ as $\Gamma_{l,m_1}, \cdots, \Gamma_{l,m_{N_d}}$ is in the descending order. Hence, $z_2 = -F_l - f_l + \sum_{n=1}^{k-1} (\Gamma_{l,m_n}) \bar{P}_{m_n}^{\text{inj}} + \Gamma_{l,m_k} (D - \sum_{n=1}^{k-1} \bar{P}_{m_n}^{\text{inj}})$. It is observed that left hand side of (12) is equal to z_2. So, based on Theorem 1, if line l is congested in the positive direction for uncertainty accommodation, then $z_2 = z_1 \ge 0$ must hold. That is, (12) must hold. □

Theorem 2 has the same identification rate as Theorem 1. The conditions in Theorem 2 represents the necessary conditions for identifying line flow constraints that may be binding. If the left hand side of (12) is negative, then the corresponding line flow constraint will not be binding.

Theorems 1 and 2 can be extended to the max-min problem with fixed unit commitment by setting $P_i = 0$, $\underline{r}_i = I_i \underline{P}_i$, and $\bar{r}_i = I_i \bar{P}_i$, $\forall i$, where I_i is the given ON/OFF indicator. They also apply to other robust/stochastic SCUC.

III. Numerical Examples

Simulations are performed for the IEEE 118-Bus system (http://motor.ece.iit.edu/data/RSCUC/IEEE_118bus.xls) with 6000 MW peak load using Gurobi 5.6.3 on PC Intel i7 3.4 GHz. The uncertainty interval bounds are all 10% of bus loads.

TABLE I
Remaining Line Constraints (24 h, %)

Load Level	20-min		30-min	
	NEG	POS	NEG	POS
100	1.57	0.69	1.77	0.81
105	1.99	0.78	2.44	0.87
110	2.42	0.87	2.82	0.94

TABLE II
Computation Benchmark for Congestion Identification

T(h)	TH1(s)	TH2(s)
24	4.4	0.08
96	17.7	0.33
168	30.34	0.52

TABLE III
Computation Benchmark for Robust SCUC (24 h)

# of Un.	PPC(s)	ORI(s)
10	13	113
20	24	>43200
30	54	>43200

Table I shows the remaining line constraints in the negative (NEG) and positive (POS) directions that may be binding based on the necessary conditions. Different load levels and response times are studied. In general, over 90% line constraints will not be binding in the max-min problem thus can be eliminated. 20-min and 30-min are the response times for uncertainty accommodation. With the increasing load and response time, more lines may be congested. Table II presents the identification performance with respect to different scheduling horizons. When $T = 168$, the identification time based on Theorem 2 (TH2) is 0.52 seconds while that for Theorem 1 (TH1) is 30.34 seconds. Table III shows the CPU time of solving the robust SCUC within column generation framework based on extreme points approach [3]. The convergence tolerance is set to 10^{-4}. The complexity of the original problem (ORI) increases dramatically with the number of uncertainties (# of Un.). When there are 30 buses with uncertainties, the ORI cannot find the solution within 12 hours (i.e. 43,200 seconds). The preprocessed model (PPC), where non-binding line flow constraints based on Theorem 2 are eliminated for the max-min subproblem, is solved in 54 seconds. The significant reduction in computation time is achieved by accelerating the max-min subproblem.

Theorem 1 and 2 could also have more potential applications besides accelerating max-min problem in robust SCUC.

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