A Systematic Method for Constructing Feasible Solution to SCUC Problem With Analytical Feasibility Conditions

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***Abstract*—Obtaining high-quality feasible solution is the core and the major difficulty in solving security-constrained unit commitment (SCUC) problems. This paper presents a systematic method for constructing feasible solutions to SCUC problem based on a group of analytical feasibility conditions. The feasibility check is performed based on the analytical necessary conditions such that most of infeasible UC states can be identified without solving LP problem. If a UC state is infeasible, it is adjusted with the possibly minimal operating cost increase based on the cost information. This UC adjusting issue is formulated as a zero-one programming problem and a branch and bound (B&B) method is established based on these feasibility conditions. Numerical testing is performed for a 31-bus system, an IEEE 24-bus system, and an IEEE 118-bus system. The testing results suggest that over 95% of infeasible UC states are identified by the analytical necessary conditions. The near-optimal feasible schedules for SCUC problem can be obtained efficiently by the proposed method. The feasible schedules obtained are compared with those obtained from mixed integer programming-based method in the IEEE 118-bus system. It is shown that the new method can produce competitive results in terms of solution quality and computational efficiency.**

***Index Terms*—Generation scheduling, Lagrangian relaxation, mixed integer programming, security constrained unit commitment.**

NOMENCLATURE

T	Number of time periods.
I	Number of thermal units.
L	Number of transmission lines.
M	Number of load buses.
t	Index for time period, $t = 1, 2, \dots, T$.
i	Index for unit, $i = 1, 2, \dots, I$.
l	Index for transmission line, $l = 1, 2, \dots, L$.
m	Index for load bus, $m = 1, 2, \dots, M$.
Γ^U	$L \times I$ matrix relating unit generations to power flows on transmission lines.
$\Gamma^U_{l,i}$	li-component of Γ^U.
Γ^D	$L \times M$ matrix relating bus loads to power flows on transmission lines.
$\Gamma^D_{l,m}$	lm-component of Γ^D.
$\overline{F}_l$	Power flow limit on transmission line l.
$D_{m,t}$	Load demand on bus m at time t.
D_t	System load demand at time t.
PR_t	Spinning reserve requirement at time t.
$\overline{P}_i$	Maximum generation level of unit i.
$\underline{P}_i$	Minimum generation level of unit i.
Δ_i	Maximum ramping rate of unit i.
$\overline{r}_i$	Maximum allowable spinning reserve contribution of unit i.
$\overline{\tau}_i$	Minimum up time of unit i.
$\underline{\tau}_i$	Minimum down time of unit i.
$S_i(\cdot)$	Total start-up/shut-down cost of unit i during the whole time horizon.
$p_{i,t}$	Power generation level of unit i at time t.
$r_{i,t}$	Spinning reserve contribution of unit i at time t.
$C_i(\cdot)$	Fuel cost of unit i at time t.
$z_{i,t}$	Discrete decision variable (status) of unit i at time t : "1" for ON and "0" for OFF.
E_{1t}	Set of ON-units at time t.
$\overline{E}_{1t}$	Set of OFF-units at time t.
E_{2t}	Set of adjustable units at time t.
E_{3t}	Set of ON-units whose ON/OFF states are unadjustable at time t.

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I. Introduction

THE aim of unit commitment (UC) is to determine the commitment states and generation levels of all generating units over a scheduling horizon to minimize the total generation cost while meeting all system-wide and individual operating constraints [1]–[3]. With the power grid operating close to the security margins in power market environment, solving security-constrained unit commitment (SCUC) problem becomes very important.

SCUC is an NP-hard mixed-integer programming problem and a number of algorithms have been developed for solving SCUC and security constrained economic dispatch (SCED) problems. The most successful and popular methods include Lagrangian relaxation (LR) [4], [5], Benders' decomposition (BD) [6], [7], mixed integer linear programming (MILP) [1], [8], and Meta-Heuristics (MH) [9], [10]. Among them, LR- and MILP-based approaches are most widely applied.

One obvious advantage of the LR-based approach is that its computational complexity of solving the dual problem is almost lineally related to the system size and therefore applicable for large scale problems. However, the solution to the dual problem is generally infeasible. That is, the once relaxed system-wide constraints may not be satisfied. Some methods, usually heuristic, are needed to construct a feasible solution to the original SCUC problem [4], [5], [11]–[13]. LR multipliers associated with spinning reserve were increased slightly in [11] and the UC problem with the new multipliers was solved to check for reserve-feasibility. A two-stage method was proposed in [12] where the discrete decision variables and the continuous decision variables were determined separately. The intermediate information, called "opportunity value", was extracted from the dual solution to unit subproblems in [4] and used to help find near optimal solutions. The relaxed UC problem was solved by the Bundle method in [5] and a priority list of units for start-up or shut-down operations was defined according to the fractional commitment state. A dynamic programming (DP) method was presented in [13] to turn ON the OFF-units when a UC state is not reserve-feasible. Nevertheless, as pointed out in [2] and [14], constructing feasible solutions with the security related constraints is very difficult for LR-based methods.

In essence, the difficulty is that the number of all possible UC states is astronomical even for a small scale problem, and feasible states take only a very small percentage of all possible UC states especially when security constraints are considered. As reported in the related literature, the basic idea for constructing feasible solutions is that if a UC state is identified as infeasible, a new one will be selected from all possible UC states and the procedure is repeated until a feasible solution is found or the computation time reaches its limit. Therefore, the efficiency of the method depends largely on the efficiency of feasibility identification and feasible UC state selection.

In the literature on LR-based approaches for SCUC, the feasibility identification of a UC state is generally accomplished by solving a linear programming (LP) problem (or quadratic programming when the fuel costs are formulated as quadratic functions) [4], [14], [15]. If an optimal solution to the LP is obtained, then the corresponding UC state is feasible and the solution to the LP is also the result of economic dispatch (ED). Otherwise, the UC state is infeasible and a new UC state is selected based on some slight adjustment to the current UC state and the procedure is then repeated. Now, it is seen that if a UC state is infeasible, the computation efforts for solving the corresponding LP problem is dispensable. The computation burden for solving the LP is heavy since the scale of the LP problem is not negligible for real scale SCUC. More seriously, if feasibility check is performed for millions of infeasible UC states before a feasible one is obtained, the overall efficiency of the method will be influenced greatly since many infeasible LP problems are solved.

Based on the above analysis, two problems must be resolved in order to improve the efficiency of the procedure for constructing feasible solutions.

1) How do we establish some fast approaches for UC feasibility check rather than solving an LP?
2) How do we obtain a feasible solution efficiently, or in other words, could some systematic and efficient method be developed for adjusting an infeasible UC state into a feasible one?

This paper tries to provide answers to the above questions. A systematic method for constructing feasible solutions to the SCUC problem is presented under LR framework. It is based on a group of analytical feasibility conditions for the SCUC problem established in our previous work [14], [16]. The basic idea is to perform feasibility check by using the analytical necessary conditions such that most of infeasible UC states can be identified without solving LP problem, and then these conditions are used in UC adjustment such that the infeasible UC state is closer to a feasible one after each adjustment (the degree of infeasibility can be measured by the total violations against system-wide constraints). Meanwhile, the "opportunity cost" is used to assure the quality of the new feasible UC state [4]. The issue of adjusting infeasible UC state is formulated as a zero-one programming problem and a branch and bound (B&B) method is established based on these feasibility conditions. Numerical testing is performed for a 31-bus system, an IEEE 24-bus system, and an IEEE 118-bus system. The testing results suggest that over 95% of infeasible UC states are identified by the analytical necessary conditions. The near-optimal feasible schedules for SCUC problem can be obtained efficiently by the proposed method. In this way, unnecessary computational requirements are avoided and the total efficiency is significantly improved. Furthermore, the new method is systematic based on the explicit and analytical conditions with solid theoretical basis.

It should be noted that MILP-based approach is becoming a mainstream method for solving SCUC problems based on commercial MILP solvers [8], [17]–[19]. However, the LR-based approaches are still useful in many cases. This is because to apply MILP-based methods efficiently, one of the most important issues is to convert the problem objective and constraints into a good linear formulation. Therefore, many ancillary variables and constraints must be introduced to handle minimum up/down time constraints, variable start-up costs, nonlinear fuel costs, etc. In some cases, the computation efficiency would be a serious issue. The approximation error is another important issue seldom discussed. On the other hand, one can use a more natural and concise formulation for the LR-based methods and cares less on the problem formation.

In fact, the new method presented in this paper for constructing a feasible solution is not only applicable for LR-based methods but also for sensitivity/perturbation analysis related issues of the SCUC problem. For example, if the system loads change slightly with the newly forecasting, a near optimal feasible schedule can be directly obtained based on the method without solving the SCUC problem. In this case, the new method is efficient for obtaining a feasible solution when performing sensitivity/perturbation analysis. It can also be used to generate an initial feasible schedule that may be needed for some MILP-based methods.

The feasible solutions obtained within LR framework are compared with those obtained from general MILP method in the IEEE 118-bus system. Numerical testing results suggest that our method can produce competitive results in terms of solution quality and could be more computationally efficient for some large problems in comparison with the commercial MILP solvers.

The rest of this paper is organized as follows. The formulation of SCUC problem is given in Section II. A concise list and interpretation of the feasibility conditions proposed in our previous work [16] are revisited for self-containing in Section III. The systematic method for constructing feasible solution is proposed in Section IV. Numerical testing results are presented and analyzed in Section V. The concluding remarks are provided in Section VI.

II. Problem Formulation

The SCUC problem considered in this paper is formulated as the following mixed-integer optimization problem:

$$\min_{p_{i,t}\ ,\ z_{i,t}} \sum_{i=1}^{I}\sum_{t=1}^{T} C_i(p_{i,t}) + \sum_{i=1}^{I} S_i(z_{i,1}\ ,\ z_{i,2}\ ,\ldots,z_{i,T}) \quad (1)$$

subject to the following constraints, in which $i=1,2,\ldots,I$, $t=1,2,\ldots,T$, and $l=1,2,\ldots,L$.

1) **System load balance**

$$\sum_{i=1}^{I} p_{i,t} = \sum_{m=1}^{M} D_{m,t} = D_t. \quad (2)$$

2) **System spinning reserve requirements**

$$\sum_{i=1}^{I} z_{i,t}\cdot \min\left\{\overline{r}_i\ ,\ \overline{P}_i - p_{i,t}\right\} \geq PR_t. \quad (3)$$

3) **Transmission line capacity (security constraints)**

$$-\overline{F}_l \leq \sum_{i=1}^{I} \Gamma_{l,i}^U p_{i,t} - \sum_{m=1}^{M} \Gamma_{l,m}^D D_{m,t} \leq \overline{F}_l. \quad (4)$$

The following notations are introduced for better presentation:

$$A = \begin{pmatrix} \Gamma_{1,1}^U & \Gamma_{1,2}^U & \cdots & \Gamma_{1,I}^U \\ \Gamma_{2,1}^U & \Gamma_{2,2}^U & \cdots & \Gamma_{2,I}^U \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{L,1}^U & \Gamma_{L,2}^U & \cdots & \Gamma_{L,I}^U \\ -\Gamma_{1,1}^U & -\Gamma_{1,2}^U & \cdots & -\Gamma_{1,I}^U \\ -\Gamma_{2,1}^U & -\Gamma_{2,2}^U & \cdots & -\Gamma_{2,I}^U \\ \vdots & \vdots & \ddots & \vdots \\ -\Gamma_{L,1}^U & -\Gamma_{L,2}^U & \cdots & -\Gamma_{L,I}^U \end{pmatrix};$$

$$B_t = \begin{pmatrix} \overline{F}_1 + \sum_{m=1}^{M} \Gamma_{1,m}^D D_{m,t} \\ \overline{F}_2 + \sum_{m=1}^{M} \Gamma_{2,m}^D D_{m,t} \\ \cdots \\ \overline{F}_L + \sum_{m=1}^{M} \Gamma_{L,m}^D D_{m,t} \\ \overline{F}_1 - \sum_{m=1}^{M} \Gamma_{1,m}^D D_{m,t} \\ \overline{F}_2 - \sum_{m=1}^{M} \Gamma_{2,m}^D D_{m,t} \\ \cdots \\ \overline{F}_L - \sum_{m=1}^{M} \Gamma_{L,m}^D D_{m,t} \end{pmatrix}$$

$$\beta = (\beta_1, \beta_2, \ldots, \beta_L, \beta_{L+1}\ldots, \beta_{2L})^T,\ \vec{1} = (1,1,\ldots,1);$$
$$A^\beta = A - \beta\cdot\vec{1},\ B_t^\beta = B_t - D_t\beta \quad (5)$$

where B_l is the minimum element of the lth row of matrix A. Based on the above notations, security constraints (4) can be equivalently transformed into the following form:

$$A^\beta(p_{1,t}p_{2,t}\ \cdots\ p_{I,t})^T \leq B_t^\beta \quad (6)$$

where A^β is a nonnegative matrix based on notations given in (5). The nonnegative coefficient matrix A^β would simplify the feasibility conditions proposed in this paper. Note that inequalities (6) are taken as the security constraints in the latter part of this paper.

4) **Generation capacity**

$$z_{i,t}\underline{P}_i \leq p_{i,t} \leq z_{i,t}\overline{P}_i \quad (7)$$

where the feasible region of discrete decision variable $z_{i,t}$ is

$$\{\, z_{i,t}\}_{i=1\,,2\,,\ldots,I}^{t=1\,,2\,,\ldots,T} \in X. \quad (8)$$

In (8), X is determined by the minimum up/down time constraints, must up/down constraints, etc. (See [4] and [14] for the detailed formulations of these constraints.)

5) **Ramp rate constraints**

$$|p_{i,t} - p_{i,t-1}| \leq \Delta_i,\ \text{ if } z_{i,t}\cdot z_{i,t-1} = 1. \quad (9)$$

The above ramp rate constraints can be simplified and included in constraint (7) implicitly. More clearly, $\overline{P}_i$ and $\underline{P}_i$

in constraint (7) for the thermal unit with ramp rate constraints can be calculated as follows:

$$\underline{P}_i = \max(\underline{P}_i, p_{i,t-1} - \Delta_i);\ \overline{P}_i = \min(\overline{P}_i, p_{i,t-1} + \Delta_i). \tag{10}$$

III. Feasibility Conditions for SCUC Problem

The following theorems given in our previous work [16] form the theoretical basis for determining the feasibility of a UC and are presented in this section for self-containing. Constraints (2), (6), and (7) at time t can be reformulated as follows.

System $P(A^\beta, B_t^\beta, D_t)$:

$$\begin{cases} \sum_{i=1}^{I} p_{i,t} = D_t \\ A^\beta (p_{1,t} p_{2.t} \ \cdots \ p_{I,t})' \le B_t^\beta \\ z_{i,t}\underline{P}_i \le p_{i,t} \le z_{i,t}\overline{P}_i, \quad i = 1,2,\ldots,I \\ z_{i,t} \in \{0, 1\}, \quad i = 1,2,\ldots,I. \end{cases} \tag{11}$$

Definition 1: A commitment state (UC) at time t

$$(z_{1,t}, z_{2,t}, \ldots, z_{I,t}) \tag{12}$$

is defined feasible to **System** $P(A^\beta, B_t^\beta, D_t)$ if and only if there exists a group of power generation levels

$$(p_{1,t}, p_{2,t}, \ldots, p_{I,t}) \tag{13}$$

such that (12) and (13) give a feasible solution to **System** $P(A^\beta, B_t^\beta, D_t)$.

It should be noted that Definition 1 is given in the form of single time period and **System** $P(A^\beta, B_t^\beta, D_t)$ does not include spinning reserve constraints (3) for convenience of analysis. Constraint (3) will be considered in Theorem 2.

A. Necessary and Sufficient Condition

Theorem 1: A commitment state $(z_{1,t}, z_{2,t}, \ldots, z_{I,t})$ at time t is feasible in the sense of Definition 1 if and only if the following SCED problem has at least one feasible solution:

$$\begin{cases} \min \sum_{i\in E_{1t}} C_i(p_{i,t}) \\ s.t. \\ \sum_{i\in E_{1t}} p_{i,t} = D_t \\ A^\beta (p_{1,t} p_{2,t} \cdots p_{I,t})^T \le B_t^\beta \\ \underline{P}_i \le p_{i,t} \le \overline{P}_i, if i \in E_{1t} \\ p_{i,t} = 0, if i \in \overline{E}_{1t}. \end{cases} \tag{14}$$

B. Analytical Necessary Conditions

Theorem 2: If a commitment state $(z_{1,t}, z_{2,t}, \ldots, z_{I,t})$ at time t satisfies spinning reserve constraints (3) and it is also feasible to **System** $P(A^\beta, B_t^\beta, D_t)$, the following inequalities must be satisfied:

$$\sum_{i\in E_{1t}} \overline{P}_i \ge D_t + PR_t \tag{15}$$

$$\sum_{i\in E_{1t}} \overline{r}_i \ge PR_t \tag{16}$$

$$\sum_{i\in E_{1t}} \underline{P}_i \le D_t \tag{17}$$

$$\sum_{i\in E_{1t}} a_{j,i}^\beta \underline{P}_i \le B_{j,t}^\beta, j = 1,2,\ldots,2L \tag{18}$$

where $a_{j,i}^\beta$ is the ji-component of A^β, $B_{j,t}^\beta$ is the j-component of B_t^β. It has been proven in [14] that the commitment state is **load-reserve feasible** if and only if inequalities (15)–(17) are satisfied. In the later part of this paper, system load constraint (2) and spinning reserve constraints (3) are always replaced by inequalities (15)–(17). Inequality (18) suggests that the minimum power flow on a transmission line should be no greater than its transmission capacity, and this inequality is a necessary condition for a commitment state to be **power-flow feasible**. Note that Theorem 2 is specific for the general type of system-wide constraints (2)–(3) and (6) for SCUC problem. If more types of system-wide constraints are taken into account, Theorem 2 is still valid since it is a necessary condition related to only a subset of the expanded system-wide constraints. Deriving the conditions associated with some new system-wide constraints is possible based on the analysis given in the paper. For example, if an emission constraint is added, a necessary condition can be derived based on the estimated lower bound on emission.

Since inequality (18) only gives a rough estimation of the minimum power flow on a transmission line when the commitment state is given, a more accurate estimation is therefore given in Theorem 3.

Theorem 3: If a commitment state $(z_{1,t}, z_{2,t}, \ldots, z_{I,t})$ at period t is feasible to **System** $P(A^\beta, B_t^\beta, D_t)$, inequality (19) must be satisfied:

$$\sum_{n=1}^{k-1} (a_{j,i_n}^\beta - a_{j,i_k}^\beta)\overline{P}_{i_n} + \sum_{n=k+1}^{N} (a_{j,i_n}^\beta - a_{j,i_k}^\beta)\underline{P}_{i_n} + a_{j,i_k}^\beta D_t \le B_{j,t}^\beta \tag{19}$$

where N is the total number of units in E_{1t}, $i_1, i_2, \ldots, i_N$ denote the order of units in E_{1t} such that

$$a_{j,i_1}^\beta \le a_{j,i_2}^\beta \le \cdots \le a_{j,i_N}^\beta \tag{20}$$

k ($1 \le k \le N$) is an integer number such that

$$\sum_{n=1}^{k-1} (\overline{P}_{i_n} - \underline{P}_{i_n}) \le D_t - \sum_{n=1}^{N} \underline{P}_{i_n} \le \sum_{n=1}^{k} (\overline{P}_{i_n} - \underline{P}_{i_n}). \tag{21}$$

Theorem 3 plays a very important role in our systematic method for constructing feasible solutions and its detailed proof is given in [16].

IV. Systematic Procedure for Constructing a Feasible Solution

For LR approach, constraint (2), (3), and (6) are initially relaxed by multipliers and the optimal dual commitment state $(z'_{1,t}, z'_{2,t}, \ldots, z'_{I,t})$ as well as optimal dual generation levels $(p'_{1,t}, p'_{2,t}, \ldots, p'_{I,t})$ are obtained at the end of dual iteration [4], [5], [11]–[13]. Since the once relaxed constraints are generally

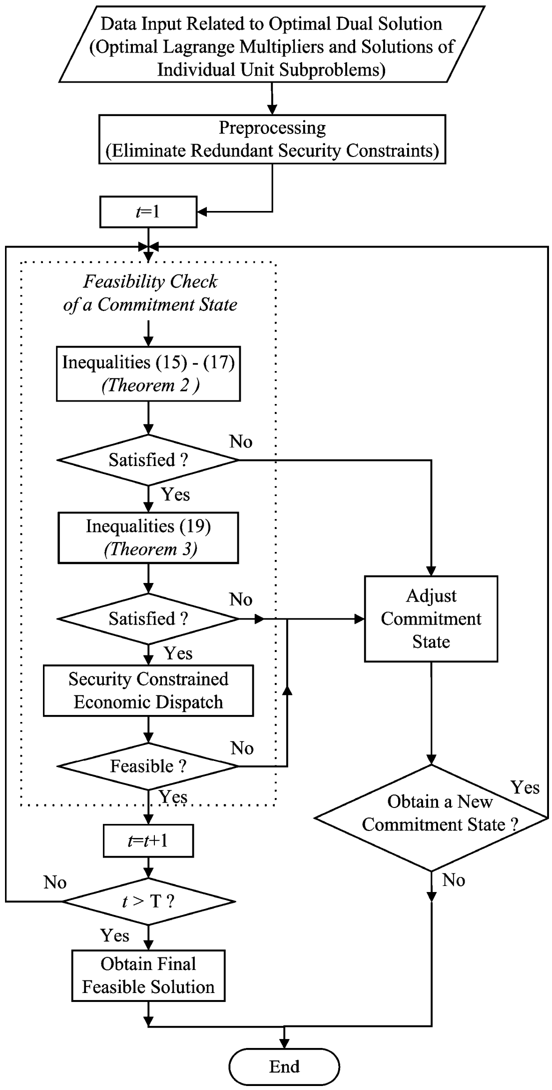

Fig. 1. Systematic method for constructing feasible solutions.

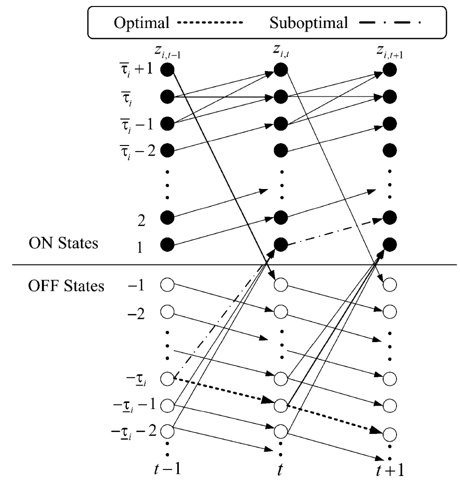

Fig. 2. State transition diagram.

not satisfied in the dual solution, we need to construct a feasible solution after the convergence of the dual solution where the subproblems for individual units are solved.

Fig. 1 shows the flowchart of our systematic method for constructing feasible solutions. Since a reduced set of security constraints would improve the computational efficiency, most of the inactive or redundant security constraints are eliminated in preprocessing stage by using the algorithm proposed in [20]. Other major steps of the method are presented below.

A. Feasibility Check of a Commitment State

As mentioned in the Introduction, the feasibility of a commitment state may have to be determined for numerous times for a large-scale SCUC problem due to combinatorial number of possible UC states. It is therefore crucial to develop a computationally efficient method for determining feasibility.

The flowchart of feasibility check for a commitment state is shown within the dotted frame in Fig. 1. According to our numerical testing results, over 95% of infeasible UC states could be identified by the analytical necessary conditions. In other words, the necessary conditions are very close to the sufficient condition. Therefore, if the inequalities (15)–(17) and (19) are satisfied by a commitment state, it is very likely to be feasible. In this case, the SCED problem is solved for this commitment state to obtain the optimal generation levels.

The advantages of applying analytical feasibility conditions are obvious. Firstly, compared with the traditional methods where the feasibility of a commitment state is unknown until the SCED problem is solved, tremendous computational efforts are saved by analytically excluding infeasible commitment states before solving the SCED problem. Secondly, if there is any violation against inequalities (15)–(17) and (19), the violation quantity can provide useful information on how to adjust the commitment state to obtain a feasible one. This will be discussed in detail in the following part.

Note that it is possible to apply this feasibility check method in B&B-based methods and MH-based approaches to accelerate the convergence. For example, a branch can be discarded if no feasible solution can be obtained along this branch based on the analytical necessary conditions. For MILP-based methods, it is possible to generate effective cutting planes based on the analytical feasibility conditions. More results will be reported in the follow-up research efforts.

B. Commitment State Adjusting

Let $(z_{1,t}^y, z_{2,t}^y, \ldots, z_{I,t}^y)$ denote commitment state obtained at the yth iteration at time t and $(z_{1,t}^0, z_{2,t}^0, \ldots, z_{I,t}^0)$ be the initial starting commitment state at this time period. If $(z_{1,t}^y, z_{2,t}^y, \ldots, z_{I,t}^y)$ is identified as infeasible, ON/OFF states of some units must be adjusted to obtain a new commitment state at the next iteration. In general, the commitment states of those units that have been ON/OFF for minimum up/down times can be adjusted without violating the minimum up/down time constraints. These units are defined as adjustable units. The ON/OFF state of an adjustable unit can be efficiently modified based on the cost information in solving individual subproblems under LR.

Fig. 2 shows the discrete state transition diagram of a thermal unit [12], [21], where each node indicates a state and each edge represents a possible state transition. The fuel cost and start-up/

shut-down costs are associated with nodes and edges, respectively. From each node at most two possible state transitions may originate as observed in Fig. 2. The optimal start-up/shut-down decisions across schedule horizon are obtained by DP as described in [21]. The ON/OFF state adjustment at time t is accomplished by giving up its optimal state transition at time period $t-1$ and reselecting another sub-optimal state transition (see Fig. 2), resulting in an increase of its total generation cost over the entire horizon, namely opportunity cost [4]. The opportunity cost is easily calculated based on the cost-to-go function in the dual solution and is an indirect indicator on how far the cost for a feasible solution maybe deviate from that of the infeasible optimal dual solution. The most obvious advantage of this method is that all individual unit constraints remain to be satisfied after the adjustment.

It should be noted that in this method, the unit states are adjusted time period by time period in sequence. Therefore, even if the start-up/shut-down decision of an adjustable unit is changed at time t, only the states after time t are changed, and the unit states at time periods from 1 to $t-1$ are kept unchanged. Therefore, the UC feasibility at earlier time periods would not be affected. If the start-up/shut-down decision of unit i is changed at time t, its initial starting states at later time periods, i.e., $z^0_{i,t+1}, z^0_{i,t+2}, \ldots, z^0_{i,T}$, should be modified accordingly. Otherwise, they are exactly the optimal dual states at later time periods, i.e., $z'_{i,t+1}, z'_{i,t+2}, \ldots, z'_{i,T}$.

With the above analysis, the problem of adjusting commitment state is formulated as the following zero-one programming problem:

$$\begin{cases} \min_{z^{y+1}_{i,t}} J = \sum_{i\in E_{2t}} f_i(z^{y+1}_{i,t}) \\ s.t. \sum_{i\in E_{2t}} z^{y+1}_{i,t}\overline{P}_i \geq D_t + PR_t - \sum_{i\in E_{3t}} \overline{P}_i \\ \sum_{i\in E_{2t}} z^{y+1}_{i,t}\overline{r}_i \geq PR_t - \sum_{i\in E_{3t}} \overline{r}_i \\ \sum_{i\in E_{2t}} z^{y+1}_{i,t}\underline{P}_i \leq D_t - \sum_{i\in E_{3t}} \underline{P}_i \\ \sum_{i_n\in E_{2t}} z^{y+1}_{i_n,t} a^{\beta}_{j,i_n} p_{i_n} \leq B^{\beta}_{j,t} - a^{\beta}_{j,i_k} D_t - \sum_{i_n\in E_{3t}} a^{\beta}_{j,i_n} p_{i_n}, j\in U_t \\ z^{y+1}_{i,t} \in \{0,1\}, i\in E_{2t} \end{cases} \tag{22}$$

where $f_i(\cdot)$ is the opportunity-cost function linearly related to $z^{y+1}_{i,t}$, the detailed formulation of which can be found in [4]; U_t is the reduced set of security constraints; N, k, and $i_1, i_2, \ldots, i_N$ are defined as in Theorem 3; and the power generation level p_{i_n} is calculated as follows:

$$\begin{cases} p_{i_n} = \overline{P}_{i_n}, if n \leq k-1 \\ p_{i_k} = D_t - \sum_{n=1}^{k-1} \overline{P}_{i_n} - \sum_{n=k+1}^{N} \underline{P}_{i_n} \\ p_{i_n} = \underline{P}_{i_n}, if n \geq k+1. \end{cases} \tag{23}$$

The problem formulated in (22) is solved by B&B algorithm, and the analytical feasibility conditions can be used to prune the B&B tree. In fact, the four inequality constraints in (22) correspond to inequalities (15)–(17) (load-reserve feasibility) and (19) (power-flow feasibility), respectively. Based on our numerical testing experiences, the first three inequality constraints are relatively easy to satisfy since adequate generation and reserve capacities are usually assured. Thus, the efficiency of the B&B algorithm depends very much on how to satisfy the remaining security constraints. This B&B algorithm is summarized in the following steps:

Step 1.1): (Initialization) Set the initial tree and obtain the initial starting commitment state $(z^0_{1,t}, z^0_{2,t}, \ldots, z^0_{I,t})$ at time t.

Step 1.2): (Obtaining upper bound) Let $i_1, i_2, \ldots, i_M$ denote the ascent order of the opportunity costs of units in E_{2t}. For $i = i_1, i_2, \ldots, i_M$, change the value of $z^0_{i,t}$. If a feasible solution can be found by simply switching the ON/OFF state of a single unit, a good upper bound UB on the objective in (22) is obtained; else, set $UB = \infty$.

Step 1.3): (Node selection) Among all live nodes, the degree of infeasibility is measured by the total violations against inequalities (15)–(17) and (19). The least violated node is chosen as an E-node.

Step 1.4): (Bound) The lower bound of the objective (22) at the E-node, denoted by LB, is determined by solving its corresponding relaxation. If $LB < UB$, mark the E-node as a live node, and go to step 1.5; else, mark it as a dead node, and then go back to step 1.3.

Step 1.5): (Branching) At the E-node, let $i_1, i_2, \ldots, i_B$ represent the order of units in E_{2t} that have not been branched such that

$$a^{\beta}_{l,i_1} \leq a^{\beta}_{l,i_2} \leq \cdots \leq a^{\beta}_{l,i_B} \tag{24}$$

where l is the index of the transmission line that is most severely overloaded. Variable $z_{i_B,t}$ is selected and branch on it. If a feasible solution to (22) is found and its objective value $J < UB$, set $UB = J$.

Step 1.6): (Pruning) Mark the E-node as a dead node if inequality (17) or (18) is not satisfied.

Step 1.7): (Stopping criterion) Repeat step 1.3–1.6 until there is no live node in B&B tree.

The optimality to problem (22) is guaranteed according to the nature of B&B algorithm. However, the optimality of this solution does not imply the optimal solution to the original problem is obtained. In adjusting UC state at time t, the unit states at all previous $t-1$ periods are kept unchanged. "Optimal" only means the opportunity cost is minimized. However, the unit states of the global optimal solution to the original problem may be different from that of the current UC state at the previous $t-1$ periods. In fact, the solution optimality generally cannot be guaranteed under LR framework but the solution quality can be measured by the duality gap.

C. Obtain Final Feasible Solution

The above procedure is applied for time $t+1$ and repeats recursively until a feasible solution over the whole schedule horizon is obtained. Though it rarely occurs in our numerical testing, it is possible that the SCUC problem is feasible but no feasible solution is found by this method. This false "infeasibility" can be improved. In fact, if a UC state is infeasible at time t, only the start-up/shut-down decisions of some certain units at time $t-1$ are adjusted in this method to make the UC state feasible at time t without affecting the feasibility of all previous $t-1$ periods. Therefore, the procedure may fail to obtain

TABLE I
GENERAL RESULTS AND STATE CHANGES AFTER ADJUSTMENT

Daily Cost = $1099489 Duality gap = 0.17% Computing Time = 2.2s	
Unit	Hours (1-24)
11	0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0
16	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0

a feasible UC state since it may be necessary to adjust the UC states at earlier periods. In this case, a feasible solution can be found if the method is revised such that the retrospective search is allowed for all previous time periods at the cost of more computational efforts.

It should be noted that the B&B method for finding feasible solution is a systematic method within integer programming framework and the rules for branching and pruning are all based on the analytical feasibility conditions rather than the heuristics and empirical results.

V. Numerical Testing

Numerical testing is performed for a 31-bus system, an IEEE 24-bus system, and the IEEE 118-bus system. The SCUC problems are solved within LR framework where the double dynamic programming method presented in [21] is applied to solve the individual subproblems and the modified subgradient method in [22] is applied to update Lagrange multipliers. The systematic method presented in this paper is used to construct feasible solutions to the original SCUC problems. The LP Optimizer in CPLEX 11.0 is utilized to solve SCED problems. The numerical testing is implemented in Microsoft Visual C# .NET on a Quad Processor PC with 4 GB RAM.

A. The 31-Bus System

The 31-bus system consists of 16 thermal generators and 43 transmission lines. The parameters of this system are given in [4]. It is found that only six transmission lines need to be considered for constructing feasible solutions [20].

By performing feasibility check for the dual commitment state obtained at the end of dual optimization, it is found that the dual commitment states at Hour 9, 10, 12, and 19 are identified as power-flow infeasible by Theorem 3. Therefore, they need to be modified. General results for this system and highlights of the ON/OFF state changes of the generators are shown in Table I. It is seen that the states of Unit 11 at Hour 10, 18 are changed to ON, and those of Unit 16 are changed to ON at Hour 9–20 as well.

In order to demonstrate the effectiveness of the analytical feasibility conditions, the entire commitment state space is completely enumerated and identified by the feasibility check method proposed in the paper. As shown in Fig. 3, over 95% of infeasible UC states are identified by the analytical necessary conditions. It is also seen in Fig. 3 that among the UC states that satisfy the analytical necessary conditions, only less than 4% of them are indentified as infeasible by running SCED, and the infeasibility of those UC states is closely related to system load profile. This will no doubt avoid much unnecessary computation requirements and significantly improve the total

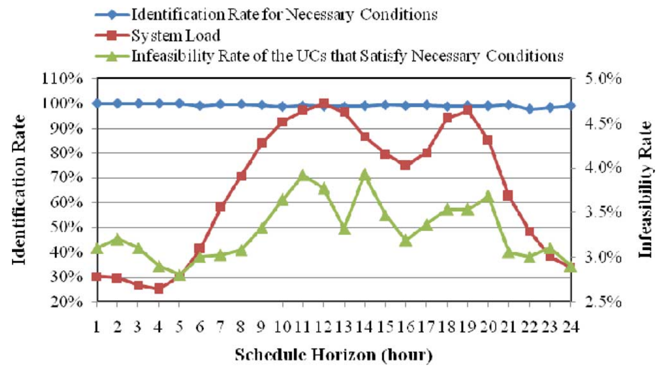

Fig. 3. Infeasibility identification by analytical conditions.

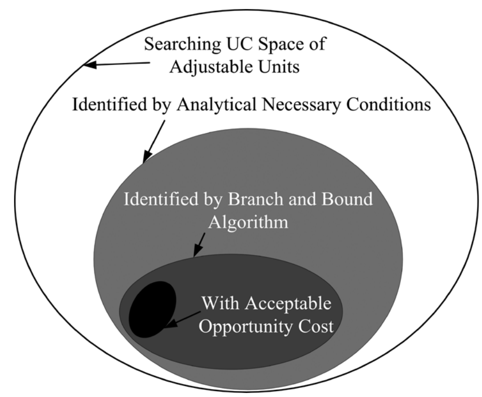

Fig. 4. Reduction of searching commitment state space.

efficiency since a large number of infeasible UC states can be excluded without solving the corresponding large-scale LP problems.

B. The IEEE 24-Bus System

The IEEE 24-bus system has 26 generators and 34 transmission lines [23]. It is found that only one transmission line is left after preprocessing [20]. We find that 99% of the infeasible UC states can be excluded by the analytical necessary conditions. The necessary conditions are very close to the sufficient conditions in this system since only one security constraint needs to be considered.

To investigate the computational savings of the proposed method, it is found that there are 421 commitment states that are not excluded by the analytical necessary conditions at the peak load hour. The benefit of the analytical feasibility condition is clear. That is, in order to find good feasible commitment states, we need only consider 421 candidates instead of 2^{26} commitment states (all units are adjustable at the peak load hour). This is a very significant improvement in computational efficiency. If we restrict total opportunity cost less than 1% of the optimal dual cost in our B&B algorithm, there will be only 17 feasible commitment states. The reduction of searching UC state space is shown in Fig. 4.

General results for this system and the changes of ON/OFF-states of the generators are listed in Table II. Compared with the results obtained in the 31-bus system, it is seen the duality gap increases from 0.17% to 0.79%. More ON/OFF-state changes

TABLE II
GENERAL RESULTS AND STATE CHANGES AFTER ADJUSTMENT

Daily Cost = $763791 Duality gap = 0.79% Computing Time = 3.8s	
Unit	**Hours (1-24)**
1	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
2	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
3	1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
5	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
10	1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
11	1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1
21	1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
22	1 1 1 1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

from the dual commitment state is the main factor leading to an increase in total opportunity cost.

C. IEEE 118-Bus System

The IEEE-118 bus system has 54 thermal generators, 186 branches, and 91 demand sides. The parameters of generators, transmission network, and load profiles are given at http://motor.ece.iit.edu/Data/. There are only 25 transmission lines left after preprocessing. In the dual solution, the number of infeasible system-wide constraints is 182 and the total amount of violations against those infeasible constraints is 6475 MW. All infeasibilities are eliminated by the B&B method after 112 iterations, and a near-optimal feasible solution for 24-h horizon with the duality gap of 0.65% is obtained. This means that a high-quality feasible schedule is constructed with the proposed method after exploring a very limited number of B&B nodes in the solution space. Moreover, after 8 cuts by the pruning conditions with negligible computational efforts, there remains at Hour 6, 9, 10–12, 20 only 64 nodes in the B&B tree instead of 2^{10} (there are 10 adjustable units at these hours) ones that are possibly feasible. Clearly there is a big savings on computational efforts.

In order to evaluate the overall performance of the LR-based method integrating our method for constructing feasible solutions, the comparative studies are conducted for the LR-based method, and MILP-based method, where the SCUC problem is solved by CPLEX 11.0 with some nonlinearity converted into linear model as in [8]. The default settings of CPLEX are selected and the maximum threshold of optimality gap is set to 0.5%. Parallel computing techniques are utilized to solve the individual subproblems in our LR implementation.

Table III lists the comparative results with the scheduling horizon ranging from 1 day to 12 days. It is seen that as the size of the SCUC problem increases, the duality gap in LR-based method is always less than 0.80%. The small duality gap in numerical testing results suggests that the proposed method performs well and the near optimal solutions are obtained. Although the gap is larger than that obtained in MILP solver, the total generation cost obtained under LR-based method for 6–12 day horizon is slightly less than that obtained in CPLEX. The computational times versus scheduling horizon for the two methods are plotted in Fig. 5. It is seen that the computational advantage of the LR-based method over the MILP-based method becomes clear as the problem size (length of scheduling horizon) increases. Computational times for constructing

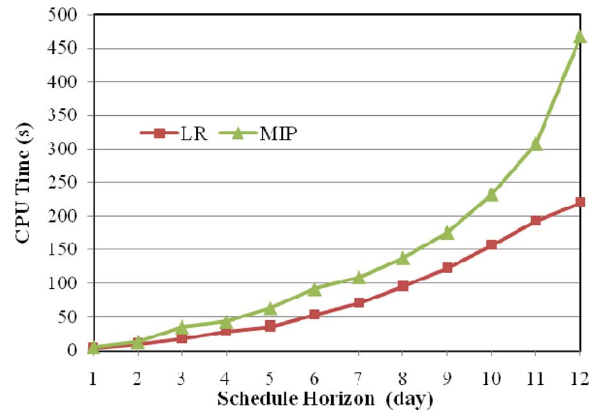

Fig. 5. CPU times versus schedule horizons.

TABLE III
COMPARISON OF LR WITH MILP FOR SOLVING SCUC PROBLEM

Horizon (days)	LR				MILP	
	CPU Time [1](s)	CPU Time [2](s)	Feasible Cost ($)	Duality Gap (%)	CPU Time [1](s)	Feasible Cost ($)
1	4.2	0.3	1640498	0.65	4.8	1640120
2	10.1	0.9	3138351	0.72	12.9	3186440
3	17.2	1.4	4735603	0.70	35.1	4733100
4	28.5	3.5	6289815	0.76	43.2	6289528
5	36.0	2.9	7805838	0.79	63.6	7805500
6	52.8	4.8	8989436	0.68	91.9	8990960
7	71.2	2.6	10137295	0.63	109.2	10138100
8	96.1	3.1	11623442	0.66	138.0	11628300
9	122.6	3.4	13270969	0.69	176.5	13277240
10	157.3	4.9	14868204	0.70	233.2	14870000
11	193.0	6.2	16424602	0.71	308.6	16429300
12	219.7	7.6	17940220	0.72	468.3	17946200

1. CPU time for solving the SCUC problem.
2. CPU time for constructing feasible solutions.

feasible solutions are listed as "CPU Time[2]" in Table III and take only a small portion of the total computational cost for solving the SCUC problem under LR-based method. The above results demonstrate the overall efficiency of our proposed method for constructing feasible solutions. It should be noted that the comparative testing results for this particular case indicate that although MILP-based approach has advantages on dealing with complicated constraints, the LR approach with the systematic method for constructing feasible solution is a good complementary method in term of computational efficiency and solution quality.

VI. CONCLUSION

This paper presents a systematic method for constructing feasible solutions to SCUC problem in LR framework. This method is based on a group of analytical conditions for determining the feasibility of the SCUC problem. The basic idea is to perform feasibility check by the analytical necessary conditions such that most of infeasible UC states can be identified and excluded without solving LPs. The infeasible commitment state is adjusted with the possibly minimal cost increase based on the opportunity cost information in the dual solution. This problem of adjusting commitment state is formulated as a 0–1 programming problem and solved by a branch and bound method with the analytical feasibility conditions incorporated for pruning and branching. The testing results show that the new method is computationally efficient and effective, and can produce competitive results in terms of solution quality and computational efficiency. Efficient cutting planes are being investigated based on the analytical feasibility conditions applied in the paper for the general MILP-based methods.

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