Robust Security-Constrained Unit Commitment and Dispatch with Recourse Cost Requirement

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***Abstract*—With increasing renewable energy resources, price-sensitive loads, and electric-vehicle charging stations in the power grid, uncertainties on both power generation and consumption sides become critical factors in the Security-Constrained Unit Commitment (SCUC) problem. Recently, worst scenario robust optimization approaches are employed to consider uncertainties. This paper proposes a non-conservative robust SCUC model and an effective solution approach. The contributions of this paper are three-fold. First, the commitment and dispatch solution obtained in this paper can be directly used in day-ahead market as it overcomes two issues, conservativeness and absence of robust dispatch, which are the two largest obstacles to applying robust SCUC in real markets. Secondly, a new concept recourse cost requirement, similar to reserve requirement, is proposed to define the upper bound of re-dispatch cost when uncertainties are revealed. Thirdly, a novel decomposition approach is proposed to effectively address the well-known computational challenge in robust approaches. Simulation results on the IEEE 118-bus system validate the effectiveness of the proposed novel model and solution approach.**

***Index Terms*—Robust SCUC, Recourse Cost, Robust Optimization, Uncertainty, Redispatch**

I. Introduction

In modern electricity markets, the commitment and dispatch of generating units are determined by solving the Security-Constrained Unit Commitment (SCUC) problem. In the U.S., the Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) run the SCUC software tool with the bid/offer data from market participants and the foretasted load information in day-ahead market (DAM) and real-time market (RTM). SCUC is a problem to find the optimal unit commitment (UC) and generation output that satisfy system constraints, such as generation load balance constraints, reserve requirements, and transmission capacity limits, as well as unit-wise constraints, such as maximum/minimum capacities, minimum on/off time requirements, and ramping up/down rate limits [1]–[4]. An assumption in conventional SCUC is that loads in the grid are known to RTOs/ISOs when SCUC is performed. However, the assumption can hardly be true in real life, especially in recent years as renewable energy sources (RES), such as wind power generation, and price-sensitive demand response result in more variations and uncertainties in power systems than ever before. Large-scale deployment of electric-vehicle charging stations also brings more uncertainties into power systems.

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If controllable generators and adjustable loads fail to follow the system condition changes, RTOs/ISOs may have to curtail the load in order to balance the system. Failure to survive from uncertainties may also jeopardize system security. In the market level, ISOs/RTOs have two chances to increase the system robustness against uncertainties from generations and loads: the SCUC solution for DAM and the SCUC solution for RTM. Considering these uncertainties and obtaining a solution that immunes the uncertainties become new and critical challenges in solving SCUC problem.

Recently, two popular approaches are applied to address the uncertainty issues. One is scenario based stochastic optimization [5]–[7], and the other is two-stage robust optimization [8]–[11]. The main idea of stochastic approach is to optimize the SCUC problem considering a set of scenarios, which are generated based on Probability Distribution Function (PDF) for uncertainties. This approach is effective when PDF is available, which, however, is not always true in practice. Another issue of this approach is that it does not guarantee the feasibility for all the uncertainties, as only limited sample points are considered. The key idea of the two-stage robust optimization is to determine the optimal UC in the first stage which leads to the least operation cost for the worst-case scenario in the second stage [8]–[10]. The solution obtained using this approach can be immunized against all possibilities of uncertainties. However, this approach is conservative because the worst-case scenario, which generally has a very low probability, is optimized. In addition, a robust dispatch that can be used by ISOs/RTOs is not specified. In order to address the conservativeness issue, authors in [12] combine the stochastic and robust approaches using a weight factor in the objective function at the cost of possible larger computation burden. To get robust dispatch solution, Affine Policy (AP) has been applied to adjust the generation levels from base dispatch in Security-Constrained Economic Dispatch (SCED) model [13], [14]. The main reason of introducing AP in robust literatures is that it convexifies the problem and makes the problem computational tractable [15]. The price of the convexification is that it shrinks the feasible set due to the strong assumption and the re-dispatch operation according to AP is non-optimal. With the strict recourse action, the multi-stage robust SCUC approaches are proposed in [16], [17]. On the other hand, the rolling optimization and Model Predictive Control (MPC) are also reported for look-ahead dispatches [13], [16], [18], [19].

In this paper, a novel robust SCUC model is proposed, which bridges the gap between robust UC and robust SCED. We optimize the operation cost for the base-case scenario only while guaranteeing the feasibility against all scenarios including the worst-case scenario. Both the UC and dispatch solutions are robust against the uncertainties. In order to limit the cost of the worst-case scenario, the Recourse Cost (RC) is proposed in this paper. RC is the re-dispatch cost for accommodating deviations from the base-case scenario. Compared to the strict AP based recourse actions in [13], [14], [16], [17], the full recourse actions are considered in the proposed model. The deliverability of ramping capabilities is also another focus on this paper in uncertainty accommodation. This paper is an extension of authors' previous work [20] with a new solution approach and more simulation results. The main contributions are listed as follows.

1) Both UC and SCED solutions are robust in the model. The upward and downward ramping capabilities are reserved at each time interval. It provides the possibility of pricing the energy and generation reserves in electricity markets within the robust framework.
2) To overcome the conservativeness issue, the base-case scenario is optimized instead of the worst-case scenario. A new concept RC requirement, similar to reserve requirement, is proposed to limit the re-dispatch cost when uncertainties are revealed.
3) Extreme point based approach is used to solve the new robust SCUC problem. By exploring the special structure of the proposed model, novel decomposition techniques are proposed in this paper to address the issues of computational tractability. By decomposing the original time-coupled problem into time-decoupled sub-problems, the solution time improves in a nonlinear fashion without loss of solution quality. It is also possible to employ parallel techniques to further reduce the solution time as the sub-problems are naturally independent. However, it is worth mentioning that the acceleration is model dependent.

The rest of this paper is organized as follows. Section II presents the problem formulation. The overall solution approach is presented in Section III, and the decomposition approach is presented in Section IV. In Section V, a comprehensive case study for the IEEE 118-bus system is conducted. We conclude the paper in Section VI.

II. Problem Formulation

A. Uncertainty Modeling

Uncertainties in power systems could come from unforeseen load demand and volatile generation output, which can be treated as negative loads. These uncertainties can be formulated as

$$\begin{aligned}\mathcal{U} := \{(\boldsymbol{\epsilon_1}, \cdots, \boldsymbol{\epsilon_T}) \in \mathbb{R}^{N_d} \times \cdots \times \mathbb{R}^{N_d} :& \\ \underline{\boldsymbol{u}}_{\boldsymbol{t}} \leq \boldsymbol{\epsilon_t} \leq \bar{\boldsymbol{u}}_{\boldsymbol{t}}, \forall t & \quad (1)\\ L_t \leq \sum_m \epsilon_{m,t} \leq U_t, \forall t\}, & \quad (2)\end{aligned}$$

where N_d and T are the numbers of uncertain load injections and scheduling periods respectively, and $\boldsymbol{\epsilon_t}$ represents the uncertainty vector at time t. $\epsilon_{m,t}$ is the entry in the vector $\boldsymbol{\epsilon_t}$. Define $\boldsymbol{\epsilon} = [\boldsymbol{\epsilon_1}^\top, \cdots, \boldsymbol{\epsilon_T}^\top]^\top$. $\mathcal{U}$ is a compact and polyhedral set (i.e., polytope). Equation (1) implies that the uncertainties are limited in intervals. The summation of these uncertainties are restricted by lower and upper bounds at time t, as shown in (2). The uncertainty set is similar to the one used in [9], and the difference is that only spatial constraint is considered in this paper. The uncertainty set can also be replaced with the one used in [8], and the techniques introduced in this paper are still applicable. Related formulation is presented in Appendix B.

B. Robust SCUC Model for Worst-case Scenario

The robust SCUC in literatures is to find the optimal UC solution that leads to the least operation cost in the worst-case scenario [8], [9]. The basic idea is consistent with [15]. For convenience, the formulation is written here as

$$\text{(P1)} \quad \min_{\boldsymbol{x} \in \mathcal{B}} \quad \left(\boldsymbol{c_b}^\top \boldsymbol{x} + \max_{\boldsymbol{\epsilon} \in \mathcal{U}} \min_{\boldsymbol{p} \in \mathcal{Q}(\boldsymbol{x}, \boldsymbol{\epsilon})} \boldsymbol{c_g}^\top \boldsymbol{p} \right), \qquad (3)$$

where $\boldsymbol{x}$ refers to the binary decisions including startup and shutdown actions and on/off indicators, and $\boldsymbol{p}$ represents the generation level vectors for units. $\mathcal{B}$ denotes the feasible set for binary variables which satisfies the constraints of startup/shutdown actions, minimum on/off time requirements for generators and so on. $\mathcal{Q}(\boldsymbol{x}, \boldsymbol{\epsilon})$ is the feasible set for dispatch given the UC $\boldsymbol{x}$ and uncertainty $\boldsymbol{\epsilon}$, which is basically a feasible set of Security-Constrained Economic Dispatch (SCED) problem. $\boldsymbol{c_b}^\top \boldsymbol{x}$ and $\boldsymbol{c_g}^\top \boldsymbol{p}$ refer to the costs associated with unit statuses and generation dispatches, respectively.

C. Robust SCUC Model with Recourse Cost Requirement

ISOs/RTOs desire to get the optimal UC and dispatch solution in the base-case scenario. They can re-dispatch the flexible resources, such as adjustable load demands, storages and generators with fast ramping capabilities, to follow the load when deviation occurs (or uncertainty is revealed). RC is basically the cost of re-dispatch with the realization of uncertainties. Let $\boldsymbol{x}$ represent all the binary variables. $\boldsymbol{p}$ and $\hat{\boldsymbol{p}}$ denote the base dispatch and the adjusted dispatch with uncertainties, respectively. It is noted that all the flexible resources are modeled as generators. The new robust SCUC model is written in abstract form as

$$\begin{aligned}\text{(P2)} \quad \psi := \min_{(\boldsymbol{x},\boldsymbol{p}) \in \mathcal{F}(c^r)} \quad & \boldsymbol{c_b}^\top \boldsymbol{x} + \boldsymbol{c_g}^\top \boldsymbol{p} & (4)\\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{p} \leq \boldsymbol{b} & (5)\\ & \boldsymbol{x} \text{ is binary vector },\end{aligned}$$

and

$$\begin{aligned}\mathcal{F}(c^r) := \Big\{ (\boldsymbol{x}, \boldsymbol{p}) : \forall \boldsymbol{\epsilon} \in \mathcal{U}, \exists \hat{\boldsymbol{p}} \text{ such that} &\\ \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\hat{\boldsymbol{p}} + \boldsymbol{E}\boldsymbol{\epsilon} \leq \boldsymbol{g} & \quad (6)\\ \boldsymbol{F}\boldsymbol{p} + \boldsymbol{G}\hat{\boldsymbol{p}} \leq \boldsymbol{\Delta} & \quad (7)\\ \boldsymbol{c_g}^\top (\hat{\boldsymbol{p}} - \boldsymbol{p}) \leq c^r \Big\}. & \quad (8)\end{aligned}$$

The main idea of the above model is to find the least cost UC and dispatch for the base-case scenario over the feasible set $\mathcal{F}(c^r)$. UC vector $\boldsymbol{x}$ and dispatch vector $\boldsymbol{p}$ in $\mathcal{F}(c^r)$ can be immunized against all possible uncertainties. $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}, \boldsymbol{E}, \boldsymbol{F}$ and $\boldsymbol{G}$ are abstract matrices for representing constraints. The RC for $(\boldsymbol{x}, \boldsymbol{p})$ is not greater than the RC requirement c^r (8).

Objective function (4) is to minimize the total operation cost for the base-case scenario, and the optimal value is denoted by ψ. Equation (5) includes all unit commitment and network constraints for the base-case scenario, the details of which can be found in [3], [4], [21]. $\mathcal{F}(c^r)$ is defined in (6-8). With the realization of uncertainties, units adjust dispatch from $\boldsymbol{p}$ to $\hat{\boldsymbol{p}}$ while respecting all the constraints (6). The redispatch from $\boldsymbol{p}$ to $\hat{\boldsymbol{p}}$ is limited by the ramping constraints (7) and RC requirement (8). Equation (7) is the abstract form for

$$\hat{P}_{i,t} - P_{i,t} \leq \Delta T R_i^u (1 - y_{i,t}), \forall i, t \quad (9)$$

$$-(\hat{P}_{i,t} - P_{i,t}) \leq \Delta T R_i^d (1 - z_{i,t+1}), \forall i, t \quad (10)$$

where $\hat{P}_{i,t}$ and $P_{i,t}$ are entries in the vector $\hat{\boldsymbol{p}}$ and $\boldsymbol{p}$, respectively. R_i^u and R_i^d are the ramping up and down rates, respectively. $y_{i,t}$ and $z_{i,t}$ are the start-up and shut-down indicators, respectively. ΔT is the response time for uncertainties. In fact, $\mathcal{F}(c^r)$ defines the reserved ramping capabilities of the system to handle uncertainties, which is similar to the reserve in the literatures [22]. A significant difference is that the deliverability of the reserve is considered in problem (P2), which also guarantees the robustness.

It should be noted that ramping constraints are modeled for $P_{i,t} \rightarrow \hat{P}_{i,t}$, $P_{i,t+1} \rightarrow \hat{P}_{i,t+1}$, and $P_{i,t} \rightarrow P_{i,t+1}$, but not for $\hat{P}_{i,t} \rightarrow \hat{P}_{i,t+1}$. That is because solutions are hourly-based in DAM, and uncertainties normally can be revealed several hours ahead in RTM or Intro-day Market (IDM). It is reasonable to assume units have enough time to re-dispatch $\hat{P}_{i,t} \rightarrow \hat{P}_{i,t+1}$ via $\hat{P}_{i,t} \rightarrow P_{i,t} \rightarrow P_{i,t+1} \rightarrow \hat{P}_{i,t+1}$ in extreme case. In the industry, this assumption is adopted during the re-dispatch process for contingency analysis. Similar assumption is also used in the scenario-based stochastic SCUC [7]. In this way, we can focus on a less conservative base-case solution that has lower cost.

Robust dispatch, which is absent in literatures [8], [9], is obtained from (P2) with robust UC simultaneously. It is important to get a robust dispatch in DAM. First, the Locational Marginal Price (LMP), used by most U.S. electricity markets, is a by-product of the SCED solution. Second, market participants need to prepare the awarded dispatches for the second day, and sometimes they may co-optimize them with other resources, such as natural gas.

The limit on RC is set in advance by ISOs/RTOs. Two possible strategies of defining RC by ISOs/RTOs are 1) Absolute RC (ARC): An absolute value; 2) Relative RC (RRC): A relative value as a percentage of the base-case cost. The RC requirement is similar to the spinning reserve requirement in the existing markets. RC is defined in terms of cost while spinning reserve is defined in terms of power. Therefore, the reserve requirement can be treated as one of the guidelines to choose RC. On the other hand, RC can also be selected proportional to the amount of the uncertainty. It is noted that the ISOs/RTOs may choose multiple RC values as candidates and calculate the optimal solutions in parallel. The total cost for base-case scenario is a monotonically decreasing function with respect to RC. A typical curve is depicted in Fig. 1. If RC is large enough, the base-case operation cost can reach its minimal value. On the contrary, if RC is too small, the base-case UC and dispatch would correspond to high operation cost, or even be infeasible. RC reflects the decision maker's degree of conservativeness to the uncertainties.

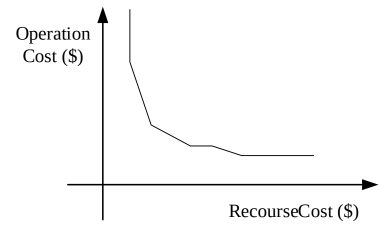

Fig. 1. Operation cost v.s. RC

D. Model Comparison

It can be observed that both (P1) and (P2) are two-stage adaptive problems. The first stage in (P1) is to find UC, and the second stage is to find the worst-case scenario. In (P2), the first stage is to find UC and dispatch for the base-case scenario, the second stages are to find the re-dispatches when the uncertainties are revealed. The adaptive problem for (P2) can be defined as

$$\begin{aligned} \min_{\hat{\boldsymbol{p}}} \quad & \boldsymbol{c}_{\boldsymbol{g}}^\top \hat{\boldsymbol{p}} \\ \text{s.t.} \quad & \boldsymbol{D}\hat{\boldsymbol{p}} \leq \boldsymbol{g} - \boldsymbol{C}\boldsymbol{x} - \boldsymbol{E}\boldsymbol{\epsilon} \\ & \boldsymbol{G}\hat{\boldsymbol{p}} \leq \boldsymbol{\Delta} - \boldsymbol{F}\boldsymbol{p} \end{aligned}$$

when $\boldsymbol{\epsilon}$ or partial $\boldsymbol{\epsilon}$ is revealed. There are two main differences.

1) Conservativeness. (P1) is conservative due to the max-min part in the objective function. It tries to minimize the worst-case cost. On the contrary, (P2) is to minimize base-case cost. Hence, solution of (P2) avoids the conservativeness issue of (P1). RC in (P2) also limits the redispatch cost for the worst-case scenario.
2) Application in practice. UC is the only usable information of (P1). A critical question regarding (P1) is how to obtain the dispatch solution which will be settled in the DAM and how to make sure that dispatch immunes against uncertainties. Even if the SCED problem is performed separately, the separation may lead to non-optimal dispatch and cause troubles for pricing energy. (P2) provides the UC and SCED solution in one shot, and the determination of energy price is possible.

In the unified approach [12], weight factor is employed to refine the conservativeness of the robust solution. In a special case where the weight factor is zero for the robust part, the robust UC with the least cost for the base case can be obtained. However, no deliverable ramping capability is specified for the ED solution obtained in this case. In comparison to this special case, a major difference is that the ED solution to (P2) is robust. In addition, the ramping capability to accommodate the uncertainties is reserved and deliverable based on the ED solution to (P2). It is also possible to calculate the reserve price within the robust model.

Algorithm 1 CG-based Procedure to Solve (P2)

$\mathcal{W} \leftarrow \emptyset, z \leftarrow +\infty$, define feasibility tolerance ϕ
while $z \geq \phi$ **do**
  Solve (MP), obtain optimal $(\boldsymbol{x}^*, \boldsymbol{p}^*)$.
  Solve (SP) with $\boldsymbol{x} = \boldsymbol{x}^*, \boldsymbol{p} = \boldsymbol{p}^*$, get solution $(z, \boldsymbol{\epsilon}^*)$
  $\mathcal{W} \leftarrow \mathcal{W} \cup \boldsymbol{\epsilon}^*$
end while

III. Solution Approach

The challenge of any robust approach is to ensure the feasibility for infinite number of constraints due to uncertainties. In comparison, traditional SCUC considers finite number of constraints. Benders Decomposition (BD) and Column Generation (CG) methods can be employed to solve (P2) exactly. Only feasibility cuts are generated in the BD method, while the point with the largest violation in the subproblem is of concern in the CG method.

The CG method in [11], which normally has better computational performance, is used to solve the model in this paper. The master problem and subproblem for the proposed model are defined as

$$\text{(MP)} \quad \min_{\boldsymbol{x},\boldsymbol{p}} \quad \boldsymbol{c_b}^\top \boldsymbol{x} + \boldsymbol{c_g}^\top \boldsymbol{p} \tag{11}$$

$$\text{s.t.} \quad \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{p} \leq \boldsymbol{b} \tag{12}$$

$$\boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{p}^k + \boldsymbol{E}\boldsymbol{\epsilon}^k \leq \boldsymbol{g}, \forall k \in \mathcal{K} \tag{13}$$

$$\boldsymbol{F}\boldsymbol{p} + \boldsymbol{G}\boldsymbol{p}^k \leq \boldsymbol{\Delta}, \forall k \in \mathcal{K} \tag{14}$$

$$\boldsymbol{c_g}^\top (\boldsymbol{p}^k - \boldsymbol{p}) \leq c^r, \forall k \in \mathcal{K} \tag{15}$$

$$\boldsymbol{x} \text{ is binary vector ,}$$

and

$$\text{(SP)} \quad z := \max_{\boldsymbol{\epsilon} \in \mathcal{U}} \quad \min_{(\boldsymbol{s_1}, \boldsymbol{s_2}, \hat{\boldsymbol{p}}) \in \mathcal{P}(\boldsymbol{\epsilon})} \boldsymbol{1}^\top \boldsymbol{s_1} + \boldsymbol{1}^\top \boldsymbol{s_2} \tag{16}$$

where

$$\mathcal{P}(\boldsymbol{\epsilon}) := \Big\{ (\boldsymbol{s_1}, \boldsymbol{s_2}, \hat{\boldsymbol{p}}) :$$

$$\boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\hat{\boldsymbol{p}} + \boldsymbol{E}(\boldsymbol{\epsilon} + \boldsymbol{s_1} - \boldsymbol{s_2}) \leq \boldsymbol{g} \tag{17}$$

$$\boldsymbol{F}\boldsymbol{p} + \boldsymbol{G}\hat{\boldsymbol{p}} \leq \boldsymbol{\Delta} \tag{18}$$

$$\boldsymbol{c_g}^\top (\hat{\boldsymbol{p}} - \boldsymbol{p}) \leq c^r \tag{19}$$

$$\boldsymbol{s_1}, \boldsymbol{s_2} \geq \boldsymbol{0} \Big\}. \tag{20}$$

$\mathcal{K}$ is the index set for uncertainty points $\boldsymbol{\epsilon}$ which are dynamically generated in (SP) with iterations. Let $\mathcal{W}$ denote the set of these uncertainty points, then $\boldsymbol{\epsilon}^k \in \mathcal{W}$. Variable $\boldsymbol{p}^k$ is associated with $\boldsymbol{\epsilon}^k$. The objective function in (SP) is the summation of non-negative slack variables $\boldsymbol{s_1}$ and $\boldsymbol{s_2}$, which evaluates the violation associated with the solution $(\boldsymbol{x}, \boldsymbol{p})$ from (MP). $\boldsymbol{s_1}$ and $\boldsymbol{s_2}$ are also explained as un-accommodated uncertainties (generation or load shedding) due to system limitations. The CG based procedure is presented in Algorithm 1. It is observed that the CG method is similar to the scenario-based method [7]. The main difference is that the sample point $\boldsymbol{\epsilon^k}$ is generated dynamically for the scenario with the largest violation.

Problem (MP) is a typical SCUC problem considering different scenarios for uncertainties. Compared with the stochastic SCUC, the number of scenarios in (MP) is limited; hence, the computational burden of (MP) is much smaller. On the contrary, problem (SP) is hard to solve, especially when the uncertainty set $\mathcal{U}$ is a general polyhedron. Problem (SP) is hard to solve since infinite uncertainty points and infinite re-dispatch strategies are considered. It is non-convex and known as computationally intractable. In [8], [9], [12], authors employed heuristic methods, such as Mountain Climbing and Outer Approximation, to solve the non-convex max-min problem. However, these methods get locally optimal value only, and have the risk of losing robustness, which is the largest merit of robust SCUC compared with stochastic SCUC. In this paper, robustness is the first priority and uncertainties must be followed in any scenario. Hence, an exact solution approach based on extreme points are used to solve (SP). However, it is computationally intractable for large systems. In Section IV, a novel decomposition approach is proposed to address this issue.

As shown in [15], [23], the optimal value to this type of max-min problem is always achieved at the extreme points of polytope $\mathcal{U}$. In this paper, another interesting explanation is presented according to the following theorem.

Theorem 1. *Let* $f(\boldsymbol{\epsilon}) := \min_{\boldsymbol{s_1}, \boldsymbol{s_2}, \hat{\boldsymbol{p}} \in \mathcal{P}(\boldsymbol{\epsilon})} \boldsymbol{1}^\top \boldsymbol{s_1} + \boldsymbol{1}^\top \boldsymbol{s_2}$. $f(\boldsymbol{\epsilon})$*, which is the minimum un-accommodated uncertainty in (SP) given* $\boldsymbol{\epsilon}$*, is a convex function of* $\boldsymbol{\epsilon}$.

The proof is presented in Appendix A. Problem (SP) can be rewritten as $\max_{\boldsymbol{\epsilon} \in \mathcal{U}} f(\boldsymbol{\epsilon})$. Since $f(\boldsymbol{\epsilon})$ is convex according to Theorem 1, its maximal value over the compact and polyhedral set $\mathcal{U}$ can always be obtained at the extreme points of $\mathcal{U}$.

According to the strong duality of Linear Programming (LP) problem, the inner minimization problem in (SP) can be converted into a maximization problem. Therefore, the max-min problem (SP) is equivalent to the following disjoint bilinear programming problem

$$\text{(SP-BI)} \quad \max_{\boldsymbol{\epsilon}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \gamma} \quad -\boldsymbol{\lambda}^\top \tilde{\boldsymbol{g}} - \boldsymbol{\lambda}^\top \boldsymbol{E}\boldsymbol{\epsilon} - \boldsymbol{\mu}^\top \tilde{\boldsymbol{\Delta}} - \gamma \tilde{c^r} \tag{21}$$

$$\text{s.t.} \quad \boldsymbol{\epsilon} \in \mathcal{U}$$

$$\boldsymbol{D}^\top \boldsymbol{\lambda} + \boldsymbol{G}^\top \boldsymbol{\mu} + \gamma \boldsymbol{c_g} = \boldsymbol{0} \tag{22}$$

$$-\boldsymbol{1} \leq \boldsymbol{E}^\top \boldsymbol{\lambda} \leq \boldsymbol{1} \tag{23}$$

$$\boldsymbol{\lambda}, \boldsymbol{\mu} \geq \boldsymbol{0}, \gamma \geq 0, \tag{24}$$

where $\tilde{\boldsymbol{g}} := \boldsymbol{g} - \boldsymbol{C}\boldsymbol{x}$, $\tilde{\boldsymbol{\Delta}} := \boldsymbol{\Delta} - \boldsymbol{F}\boldsymbol{p}$ and $\tilde{c^r} := c^r + \boldsymbol{c_g}^\top \boldsymbol{p}$ are constants, as $\boldsymbol{x}$ and $\boldsymbol{p}$ are solutions to (MP). $\boldsymbol{\lambda}, \boldsymbol{\mu}$, and γ are Lagrangian multipliers for (17-19),

As stated previously, the optimal $\boldsymbol{\epsilon}$ to (SP-BI) are the extreme points of $\mathcal{U}$. Hence, candidates of optimal $\boldsymbol{\epsilon}$ can be written as a function of the extreme points directly by introducing auxiliary binary variables when closed form of extreme points are available. In this way, the original infinite set of continuous variable $\boldsymbol{\epsilon}$ is reduced to finite extreme points of $\mathcal{U}$. Consider an example as follows. Candidates of optimal

$\boldsymbol{\epsilon}$ can be written as

$$\begin{cases} \boldsymbol{\epsilon} = \sum_n y_n \boldsymbol{\epsilon}_n^e, & (25) \\ \sum_n y_n = 1,\ y_n \in \{0,1\}, \forall n, & (26) \end{cases}$$

where $\boldsymbol{\epsilon}_n^e$ is the n^{th} extreme point for $\mathcal{U}$, and it is a constant vector. y_n is the indicator of $\boldsymbol{\epsilon}_n^e$ being selected as optimal $\boldsymbol{\epsilon}$. It is noted that (25-26) is just an example and the general formulation for typical budget sets is shown in Appendix B.

The only quadratic term $\boldsymbol{\lambda}^\top \boldsymbol{E}\boldsymbol{\epsilon}$ in (SP-BI) is linearized as follows. Let $\boldsymbol{\beta} := \boldsymbol{\lambda}^\top \boldsymbol{E}$, which has the same dimension as $\boldsymbol{\epsilon}$. Then, we have

$$\boldsymbol{\lambda}^\top \boldsymbol{E}\boldsymbol{\epsilon} = \sum_m \beta_m \epsilon_m = \sum_m \sum_n \beta_m y_n (\boldsymbol{\epsilon}_{\boldsymbol{n}}^{\boldsymbol{e}})_m, \qquad (27)$$

where subscript m denotes the m^{th} entry in vector. Let $w_{mn} := \beta_m y_n$, then

$$w_{mn} = \begin{cases} \beta_m, & \text{if } y_n = 1 \quad (28) \\ 0, & \text{if } y_n = 0. \quad (29) \end{cases}$$

Since $-1 \leq \beta_m \leq 1$ according to (23), $-1 \leq w_{mn} \leq 1$ holds. Therefore, the linear form of the above equations is

$$\begin{cases} w_{mn} \geq \beta_m + y_n - 1, \quad w_{mn} \geq -y_n & (30) \\ w_{mn} \leq \beta_m + 1 - y_n, \quad w_{mn} \leq y_n. & (31) \end{cases}$$

A merit of (30-31) is that the coefficients for binary variables y_n are 1 or -1 instead of big-M in literatures. This is important in terms of computational tractability. With the exact linearization, problem (SP-BI) is converted into an MILP problem

$$\text{(SP-E)} \max_{\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\beta},\gamma,w_{mn}} -\boldsymbol{\lambda}^\top \tilde{\boldsymbol{g}} - \sum_m \sum_n w_{mn}(\boldsymbol{\epsilon}_n^e)_m - \boldsymbol{\mu}^\top \tilde{\boldsymbol{\Delta}} - \gamma \tilde{c}^r$$
$$\text{s.t.} \quad (22-24), (26), (30-31)$$
$$\boldsymbol{\beta} = \boldsymbol{\lambda}^\top \boldsymbol{E} \qquad (32)$$

It is noted that, depending on the sign of $(\boldsymbol{\epsilon}_n^e)_m$, only one of (30-31) is necessary and the other is redundant for w_{mn}.

IV. Relax-and-enforce Decomposition Approach

CG based framework is employed to solve the robust SCUC problem in this paper as shown in Algorithm 1. In Section III, a linearized model (SP-E) to solve problem (SP) exactly is presented, but it is computationally intractable. As shown in Section II-A, uncertainty $\boldsymbol{\epsilon}$ is time decoupled. So, even if there are only 10 different extreme $\boldsymbol{\epsilon_t}$ at time t, then 10^{24} extreme points should be considered in (SP-E) for 24-hour scheduling problem. The computational burden is extremely large for the modern MILP solvers.

In this section, novel decomposition techniques are proposed to accelerate the solution process. The decomposition approach is employed to solve the problem (SP) efficiently within the CG-based framework. The basic idea is to decompose the original time-coupled problem (SP) into smaller time-decoupled subproblems for individual time intervals, and then solve them separately. In comparison to other models [8], [10], another merit of the proposed model is that it has special structure in (SP). $\boldsymbol{s_1}$, $\boldsymbol{s_2}$, and $\hat{\boldsymbol{p}}$ in (SP) are time-coupled due to the RC requirement constraint (19). Fortunately, this is also the only constraint coupling the variables in time dimension. It is popular to employ Lagrangian Relaxation to decouple variables in SCUC literatures [1], [4]. However, it cannot be used to relax (19), due to the large gap between the original and the relaxed problem. In this paper, we address the time coupling issue exactly by introducing a RC searching problem.

First, we formulate a new problem (SP-1) as follows.

$$\text{(SP-1)} \quad z_1 := \max_{\boldsymbol{\epsilon}\in\mathcal{U}} \min_{(\boldsymbol{s_1},\boldsymbol{s_2},\hat{\boldsymbol{p}})\in\mathcal{P}_1(\boldsymbol{\epsilon})} \boldsymbol{1}^\top \boldsymbol{s_1} + \boldsymbol{1}^\top \boldsymbol{s_2}$$

where

$$\mathcal{P}_1(\boldsymbol{\epsilon}) := \Big\{(\boldsymbol{s_1}, \boldsymbol{s_2}, \hat{\boldsymbol{p}}) : \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\hat{\boldsymbol{p}} + \boldsymbol{E}(\boldsymbol{\epsilon} + \boldsymbol{s_1} - \boldsymbol{s_2}) \leq \boldsymbol{g}, \; \boldsymbol{F}\boldsymbol{p} + \boldsymbol{G}\hat{\boldsymbol{p}} \leq \boldsymbol{\Delta}, \; \boldsymbol{s_1}, \boldsymbol{s_2} \geq \boldsymbol{0}. \Big\}.$$

Compared with $\mathcal{P}(\boldsymbol{\epsilon})$ in problem (SP), RC constraint (19) is relaxed in set $\mathcal{P}_1(\boldsymbol{\epsilon})$. Given base-case UC $\boldsymbol{x}$ and dispatch $\boldsymbol{p}$, the optimal value z_1 to (SP-1) is the un-accommodated uncertainty. It is an underestimation as RC requirement is ignored.

Second, another new problem is formulated as

$$\text{(SP-2)} \quad z_2 := \max_{\boldsymbol{\epsilon}\in\mathcal{U}} \min_{\hat{\boldsymbol{p}}\in\mathcal{P}_2(\boldsymbol{\epsilon})} \boldsymbol{c_g}^\top(\hat{\boldsymbol{p}} - \boldsymbol{p}) - c^r$$

where

$$\mathcal{P}_2(\boldsymbol{\epsilon}) := \Big\{\hat{\boldsymbol{p}} : \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\hat{\boldsymbol{p}} + \boldsymbol{E}\boldsymbol{\epsilon} \leq \boldsymbol{g}, \quad \boldsymbol{F}\boldsymbol{p} + \boldsymbol{G}\hat{\boldsymbol{p}} \leq \boldsymbol{\Delta}\Big\}.$$

It is observed that no slack variable is used in (SP-2). The objective is to find the $\boldsymbol{\epsilon}$ leading to the largest violation of RC requirement. Given the solution $(\boldsymbol{x}, \boldsymbol{p})$ to problem (MP), we can solve the problem (SP-1) and (SP-2) sequentially. And we have the following proposition regarding the solutions to (SP-1) and (SP-2).

Proposition 1. *Given base-case UC $\boldsymbol{x}$ and dispatch $\boldsymbol{p}$, if the optimal value to problem (SP-1) is zero, then problem (SP-2) is feasible, and the RC requirement violation is z_2. If $z_2 \leq 0$, then $(\boldsymbol{x}, \boldsymbol{p})$ is feasible to (P2).*

It can be observed that variables in both problem (SP-1) and (SP-2) are time-decoupled. Therefore they can be decomposed into subproblems for individual time intervals. We add subscript t to denote the variables and parameters for the subproblems at time t in the following context. For example, $\boldsymbol{x}$ is denoted as $[\boldsymbol{x_1}^\top \cdots \boldsymbol{x_t}^\top \cdots \boldsymbol{x_T}^\top]^\top$. Subproblems of (SP-1) and (SP-2) at time t can be formulated as

$$\text{(SP-1-t)} \quad z_{1,t} := \max_{\boldsymbol{\epsilon_t}\in\mathcal{U}_t} \min_{(\boldsymbol{s_{1,t}},\boldsymbol{s_{2,t}},\hat{\boldsymbol{p}}_t)\in\mathcal{P}_{1,t}(\boldsymbol{\epsilon_t})} \boldsymbol{1}^\top \boldsymbol{s_{1,t}} + \boldsymbol{1}^\top \boldsymbol{s_{2,t}}$$

where

$$\mathcal{P}_{1,t}(\boldsymbol{\epsilon_t}) := \Big\{(\boldsymbol{s_{1,t}}, \boldsymbol{s_{2,t}}, \hat{\boldsymbol{p}}_t) : \boldsymbol{C_t}\boldsymbol{x_t} + \boldsymbol{D_t}\hat{\boldsymbol{p}}_t + \boldsymbol{E_t}(\boldsymbol{\epsilon_t} + \boldsymbol{s_{1,t}} - \boldsymbol{s_{2,t}}) \leq \boldsymbol{g_t}, \; \boldsymbol{F_t}\boldsymbol{p_t} + \boldsymbol{G_t}\hat{\boldsymbol{p}}_t \leq \boldsymbol{\Delta_t}, \; \boldsymbol{s_{1,t}}, \boldsymbol{s_{2,t}} \geq \boldsymbol{0}\Big\},$$

and

$$\text{(SP-2-t)} \quad z_{2,t} := \max_{\boldsymbol{\epsilon_t}\in\mathcal{U}_t} \min_{\hat{\boldsymbol{p}}_t\in\mathcal{P}_{2,t}(\boldsymbol{\epsilon_t})} \boldsymbol{c_t}^\top \hat{\boldsymbol{p}}_t$$

Algorithm 2 Decomposition Procedure to Solve (P2)

```
W ← ∅, z_1 ← +∞, z_2 ← +∞, define tolerance φ
while z_1 ≥ φ or z_2 ≥ φ do
    Solve (MP), obtain optimal (x*, p*).
    z_1 ← 0
    for t = 1 to T do
        Solve (SP-1-t) with x_t = x*_t, p_t = p*_t, obtain optimal (z_{1,t}, ε*_t)
        z_1 ← z_1 + z_{1,t}
    end for
    if z_1 < φ then
        z_2 ← 0
        for t = 1 to T do
            Solve (SP-2-t) with x_t = x*_t, p_t = p*_t, obtain optimal (z_{2,t}, ε*_t)
            z_2 ← z_2 + z_{2,t}
        end for
        z_2 ← z_2 − c^r − c_g^⊤ p*
    end if
    ε* ← [ε*_1^⊤, ··· , ε*_T^⊤]^⊤
    W ← W ∪ ε*
end while
```

where

$$\mathcal{P}_{2,t}(\boldsymbol{\epsilon_t}) := \Big\{ \boldsymbol{\hat{p}_t} : \boldsymbol{C_t x_t} + \boldsymbol{D_t \hat{p}_t} + \boldsymbol{E_t \epsilon_t} \leq \boldsymbol{g_t} \quad \boldsymbol{F_t p_t} + \boldsymbol{G_t \hat{p}_t} \leq \boldsymbol{\Delta_t} \Big\}.$$

The approach to solve (SP) presented in Section III is still applicable to subproblems (SP-1-t) and (SP-2-t). The time-decoupled procedure to solve problem (P2) is presented in Algorithm 2.

Proposition 2. *The optimal value to (MP) in Algorithm 2 is always a lower bound of the optimal value to problem (P2).*

Theorem 2. *Given compact and polyhedral set $\mathcal{U}$ defined in (1-2), the procedure in Algorithm 2 converges in finite steps, and the optimal $(\boldsymbol{x^*}, \boldsymbol{p^*})$ to (P2) is obtained when the procedure converges.*

See Appendix C for the proof. According to the simulation experiences, the procedure normally converges after several iterations. By employing Algorithm 2, the combinatorial explosion issue is effectively addressed. Considering the example at the beginning of this section, instead of 10^{24} extreme points, only 10 extreme points are modeled in (SP-1-t) or (SP-2-t) at each time interval, and the optimal value to (SP-1) or (SP-2) can be obtained by solving 24 similar subproblems. The original feasibility check problem (SP) is decomposed into two steps, where the first step is to check the generation or load shedding by relaxing the resource cost constraint and the second step is to check the recourse cost violation until the recourse constraint is enforced. We call the proposed approach *relax-and-enforce* decomposition approach. It should be noted that the acceleration techniques are model dependent. They are not applicable to the models in [8], [10].

Remark 1. *Parallel computing can be employed to solve the problem (SP-1-t) and (SP-2-t) simultaneously for all time intervals. These problems are naturally independent with each other, hence the total solution time can be reduced considerably by parallel computing.*

Remark 2. *According to the Proposition 1, the tightest upper bound of the RC for the optimal $(\boldsymbol{x}, \boldsymbol{p})$ to (P2) can be obtained as a byproduct, which can be used to help the decision marker determine a proper value of RC requirement.*

TABLE I
BASE COST SENSITIVITY ANALYSIS WITH RC REQ.*

c^r(k\$)	Iter.	Base Cost (\$)	Worst Cost (\$)	UCs(h)
102	5	1,934,367	2,036,367	771
112	5	1,924,318	2,036,318	772
122	5	1,914,426	2,036,426	770
132	5	1,904,893	2,036,893	772
142	5	1,895,525	2,037,525	767
152	5	1,886,402	2,038,402	763
162	5	1,877,659	2,039,659	762
172	5	1,869,490	2,041,490	751
182	5	1,862,697	2,044,697	746
192	3	1,859,925	2,050,249	738
202	3	1,859,925	2,050,249	738

* $\alpha = 30\%, \beta = 0.85$

V. CASE STUDY

Numerical testing is performed on the modified IEEE 118-bus system with 54 thermal units and 186 branches. The new robust SCUC problem is solved using procedures proposed in this paper. MILP problems are solved by Gurobi 5.6.3 [24] on PC with Intel i7-3770@3.40GHz 8GB RAM. In Section V-A, the operation costs for base-case scenario are reported for different levels of RC. Comparisons between RC requirement and reserve requirement are presented in Section V-C. Finally, we report computational benchmark testing results for different approaches in Section V-D.

The peak load is 6600MW in 24 hours for the modified IEEE 118-bus system. The detailed data including generator parameters, line reactance and ratings, and load profiles can be found at http://motor.ece.iit.edu/Data/ROSCUC_118.xls. The uncertainty $\boldsymbol{\epsilon_t}$ at time t respects $-\alpha_t \boldsymbol{d_t} \leq \boldsymbol{\epsilon_t} \leq \alpha_t \boldsymbol{d_t}$, and $L_t \leq \mathbf{1}^\top \boldsymbol{\epsilon_t} \leq U_t$, where $\boldsymbol{d_t}$ denotes the foretasted load vector at time t for the buses with uncertainties, $L_t = -\beta_t \alpha_t \mathbf{1}^\top \boldsymbol{d_t}$, and $U_t = \beta_t \alpha_t \mathbf{1}^\top \boldsymbol{d_t}$. Parameters $\alpha_t \in \mathbb{R}$ and $\beta_t \in [0, 1]$ represent degrees of uncertainty at bus levels and system level, respectively.

A. Operation Cost v.s. RC Requirement

It is assumed that load uncertainties are located in 10 buses out of the 91 load buses, where the loads are greater than 1.95% of the system load. The α_t is set to 30%, and the β_t is set to 0.85 for all time periods. The RC levels are adjusted for the sensitivity analysis. We set the feasibility tolerance $\phi = 0.001$ and use the default settings in Gurobi.

Table I presents simulation results with increasing levels of RC requirement c^r. The second column shows the iteration number of the approach. The third and fourth columns list the

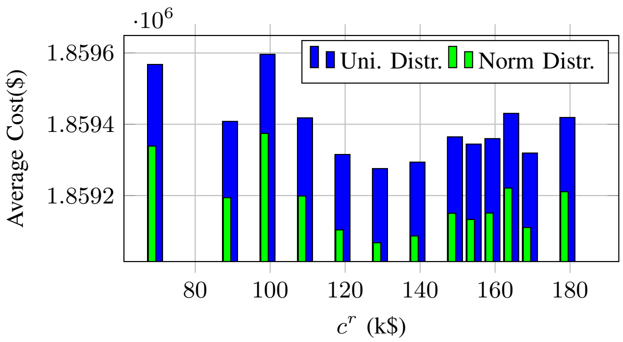

Fig. 2. Average Cost with Different RCs

operation costs in base-case and worst-case respectively. The worst-case cost is obtained by adding the tightest upper bound of RC to base-case cost. Total unit commitment hours, which are the summations of committed unit-hours, are presented in the last column. It can also be used to distinguish different UC solutions for diverse c^r. It should be noted that both the dispatches and UCs in this paper are robust against the uncertainties which are different from the robust UCs only in literatures [8], [9], [12], where the SCED problem is performed with full 24-hour load information once the robust UC solution is obtained. In our experiment, the robust dispatch is determined without exact load information. It is observed that, the base-case cost decreases with the increment of the RC requirement. This is consistent with the curve depicted in Fig. 1. The lower bound of the base-case cost is achieved when $c^r \geq \$192,000$ and the tightest upper bound of RC is $\$190,324$, which is a byproduct of Algorithm 2.

The data in Table I also demonstrates that if the *worst-case cost* is employed as the objective function, then it leads to conservative UC and dispatch solutions for the base case. Lower worst-case cost normally causes higher base-case cost. In Table I, the lowest worst-case cost $\$2,036,318$ is obtained when $c^r = \$112k$ with the 2nd largest base-case cost $\$1,924,318$. On the contrary, when $c^r = \$172k$, base-case cost is decreased by 2.85% ($54,828) while the worst-case cost is increased by only 0.25% ($5,172). In this example, $\$172k$ is consider as a better RC requirement than $\$112k$, since we can save much more money in the base-case by paying a little more if the rare worst-case occurs. The changes of base-case and worst case costs are nonlinear with the increment of c^r. For example, the base-case cost increases by 0.365% when changing c^r from $182k to $172k. The base-case cost increases by 0.484% when replacing c^r $152k with $142k. It is observed that it becomes expensive to lower the c^r if it is already small.

In general, the robustness can always be guaranteed as long as the system has large enough ramping capability. The ramping capability is provided by the online units in our experiment. However, it is economically inefficient to have too many unnecessary units online. As shown in the last column of Table I, extra generators are committed with the lowest worst-case cost if c^r is too low. This is the issue of robust SCUC model in the existing literatures. Using the model proposed in this paper, much less conservative options in Table I are available to the decision maker.

The average operation costs for different choices of RCs are shown in Fig. 2. Both normal distribution and uniform distribution are tested. They are calculated from 1000 randomly generated load points, given the base-case UC and dispatch solutions. Fig. 2 shows that the average cost is dependent on the PDF of uncertainty and nonlinear with respect to c^r. In this example, the lowest cost is obtained at $c^r = \$132k$, although variations of average cost are small. An observation is that the average cost obtained with normal distributed uncertainty is lower than that with uniform distributed uncertainty.

TABLE II
AVERAGE COST FOR BASE-CASE AND WORST-CASE ORIENTED MODELS ($\beta = 0.8$)

α	Base-Case Oriented ($)	Worst-Case Oriented($)	Difference ($)
5	1,763,458.39	1,768,389.05	4,930.67
4	1,760,138.73	1,764,623.7	4,484.97
3	1,757,905.63	1,760,625.46	2,719.83
2	1,756,610.09	1,758,117.48	1,507.39

TABLE III
AVERAGE COST FOR BASE-CASE AND WORST-CASE ORIENTED MODELS ($\alpha = 3$)

β	Base-Case Oriented ($)	Worst-Case Oriented($)	Difference ($)
0.9	1,758,593.52	1,762,390.13	3,796.61
0.8	1,757,905.63	1,760,625.46	2,719.83
0.7	1,757,323.87	1,758,907.76	1,583.89
0.6	1,756,969.04	1,757,966.32	997.28

B. Base-Case V.S. Worst-Case Oriented Model

In the existing robust UC literatures, the worst-case scenario is optimized. Both the robust UC and robust ED solution can be obtained in a shot in the model proposed in this paper. As the robust ED is absent in existing robust SCUC models [8]–[10], [12], it would be unfair to compare them directly. Therefore, we perform the simulations using the model proposed in this paper but with different objective functions. In the base-case oriented model, we formulate the base-case cost as the objective function. In the worst-case oriented model, the worst-case cost is formulated as the objective function. Both robust UC and robust ED can be obtained in the simulations. Hence, we can compare the conservativeness of the base-case and worst-case oriented models. In this subsection, uncertainties are assumed located at buses 17, 19, 40, 60, and 90. As the number of buses with uncertainty is decreased, we increase the uncertainty amount at each bus (i.e. α). For simplicity, the recourse cost requirement is dropped (i.e., $c^r = +\infty$). The experiments are performed as follows: 1) solve the base-case oriented and worst-case oriented robust SCUC problem with different uncertainty parameters α and β, then obtain two sets of base solution; 2) randomly generate 500 samples assuming normal distribution for uncertainties, and calculate the average cost.

TABLE IV
AVERAGE COST AND FEASIBILITY COMPARISON $\alpha = 0.85$

β	Average Cost ($) Rob.	Average Cost ($) Res.	Infeasibility (%) Rob.	Infeasibility (%) Res.	Base Cost ($) Rob.	Base Cost ($) Res.
0.65	1,751,609	1,775,124	0	7.94	1,751,723	1,748,361
0.7	1,752,020	1,775,043	0	7.95	1,752,081	1,748,361
0.75	1,752,541	1,775,043	0	7.94	1,752,721	1,748,361
0.8	1,753,051	1,775,043	0	7.94	1,753,403	1,748,361

TABLE V
AVERAGE COST AND FEASIBILITY COMPARISON $\alpha = 0.75$

β	Average Cost ($) Rob.	Average Cost ($) Res.	Infeasibility (%) Rob.	Infeasibility (%) Res.	Base Cost ($) Rob.	Base Cost ($) Res.
0.6	1,749,988	1,762,201	0	5.05	1,749,810	1,748,361
0.7	1,750,562	1,762,531	0	5.03	1,750,389	1,748,361
0.8	1,751,321	1,762,539	0	5.02	1,751,117	1,748,361

The experiment results are presented in Table II and Table III. In Table II, the β is fixed to 0.8. The average costs for both the base-case and worst-case oriented models vary with the choices of α. The largest average cost is $1,768,398 which is obtained when $\alpha = 5$ in the worst-case oriented model. In contrast, the average cost is $1,763,458 for the base-case oriented model, which is $4,930 cheaper than that for the worst-case oriented model. It can be observed that the average cost decreases with the decrement of α. For the same α, the average cost for the base-case oriented model is always smaller than that for the worst-case oriented model. Another observation is that the average cost difference for the two models is also an increasing function with respect to α.

In Table III, the α is fixed to 3, and β varies. It means that bounds of the uncertainty at individual buses remain the same while the system-wide uncertainty changes. The smallest average cost is obtained in the base-case oriented model when $\beta = 0.6$. Table III shows that average costs obtained in both base-case and worst-case oriented models is an increasing function with respect to β. The difference of the average cost for these two models also increases with the increment of the system-wide uncertainty. In this subsection, it can be observed that the base-case oriented model is less conservative than the worst-case oriented model.

C. Comparisons with Reserve Requirement

In the U.S. electricity markets, a certain level of reserve services are normally maintained. Spinning reserve can provide required ramping capability in the system for accommodating the variations of loads and generations. However, load following is not always guaranteed by these reserve requirements. Due to the network congestions, the ramping capability sometimes cannot be delivered to the buses with uncertain loads. In this part, uncertainties are assumed for loads at buses 15, 54, 59, and 80. The RC requirement is relaxed. The experiments are performed as follows, 1) solve the conventional SCUC problem with 10% spinning reserve requirement and the proposed robust SCUC model, then obtain two sets of base solution; 2) randomly generate 2000 scenarios assuming normal distribution for uncertainties, and check the feasibility of each scenario and calculate the average cost of re-dispatch for all scenarios. A scenario is feasible for the base solution when generation/load shedding is not required.

Table IV presents the simulation results where $\alpha = 0.85$ and the penalty for infeasibility (generation/load shedding) is $5000/MWh. Uncertainty degree increases with the value of β. It can be observed that the average cost and base-case cost for robust approach, column "Rob.", are very close to each other. In this case, we can estimate the average cost from the base-case cost, which is useful in practice. Note that this conclusion is obtained under the assumption that load deviations follow a normal distribution. The average cost for the robust approach increases with the increment of β. It makes sense as bigger β means larger set for uncertainties, which makes the feasible set of (P2) smaller and leads to higher base-case cost. On the contrary, the base-case and average cost of SCUC with reserve requirement, column "Res.", barely changes with β. Another important result from Table IV is the robustness comparison between the robust approach and the reserve approach. It can be observed that the unit can always be re-dispatched to follow the uncertainties from base-case dispatch solutions in the robust approach. However, the dispatch solutions in the reserve approach cannot immune against all uncertainties. About 7.94% scenarios are infeasible when conventional SCUC is employed. Due to the high penalty of the unfollowed uncertainty (generation or load shedding), the average cost of the reserve approach is around 1.28% higher than that of the robust approach proposed in this paper.

Table V shows the results for $\alpha = 0.75$. The interval of the uncertainty is reduced. It can be observed that the average costs of both approaches are lowered. The infeasible scenarios in the reserve approach is also reduced to 5.02% from 7.94%. By comparing the data in Table V and Table IV, an interesting observation is that the infeasibility rate is not related to the aggregated uncertainty value (i.e. β), but with the uncertainty on individual buses (i.e. α). It indicates that the total ramping capability provided in the system is sufficient in these cases, but due to transmission line congestion, the ramping capability cannot be delivered to the desired buses.

D. Computational Burden Comparisons

In this part, computational burdens are compared for the original approach (the approach in Section III) and the decomposition approach. Due to the large solution time for the original approach, "TimeLmit" is set as 3600s and "MIPGap" is set as 0.005 in GUROBI. Uncertainties are considered at 10 buses, and the SCUC problems for 1 Day, 2 Day, and 3 Day are simulated in this part. The solution times and the base-case costs are presented in Table VI. According to the experiments, more than 95% of the computation time is spent on solving the max-min problem (SP). Simulations are terminated due to the "TimeLimit" when solving the single MILP problem (SP) in the original approach. Hence, the CPU times "OriTime" for solving the entire problems are close and large for 1 Day, 2 Day, and 3 Day. In comparison, simulations are terminated as

TABLE VI
CPU TIME COMPARISON

T(h)	DecTime (s)	OriTime (s)	DecCost ($)	OriCost ($)	UnFo (MW)
24	67	11,328	1,901,851	1,901,851	0
48	151	10,911	3,806,850	3,806,787	0.4
72	580	11,007	5,498,230	5,498,041	0.81

the gap limits are reached in the decomposition approach. The CPU times "DecTime" are much less than "OriTime". It can be further reduced if parallel computing techniques are employed. Since the gaps are not reached in the original approach, the solution quality cannot be guaranteed. It is observed that costs "OriCost" obtained by the original approach are smaller than costs "DecCost" obtained by the decomposition approach for 2-Day and 3-Day problems. In other words, the (MP) in the original approach is a relaxed problem of that in the decomposition approach. It indicates that the UC and dispatch solution from the original approach cannot immune against all the uncertainties while respecting the re-dispatch constraints. This is the adverse impact of losing gap guarantee on solving (SP). The last column UnFo shows the largest unfollowed uncertainty for the original approach.

VI. CONCLUSION

A novel non-conservative robust SCUC model is proposed in this paper. Instead of considering the worst-case scenario, the base-case scenario is optimized while ensuring that the UC and dispatch solution can immune against all the uncertainties. A new concept RC requirement is proposed to define the upper bound of the redispatch cost when uncertainties are revealed. Extreme points based solution approach is used to solve the problem. A decomposition approach is proposed to accelerate the solution process. Simulations on the IEEE 118-bus system demonstrate the effectiveness of the model and solution approach. Most importantly, the two largest obstacles, conservativeness and absence of robust dispatch, to applying robust SCUC in real markets are removed in this paper. In addition, the computational challenge for the proposed robust SCUC model is also effectively addressed by the proposed decomposition approach. Accordingly, it is possible to apply the proposed robust SCUC model and the associated solution approach in real electricity markets.

The research on the robust SCUC and dispatch with known PDF information is ongoing. In this case, the objective function can be replaced with the expected cost. In this paper, the acceleration is achieved by Relax-and-Enforce decomposition. The computation burden of (SP-1-t) or (SP-2-t) for a single time interval could still be large if the number of extreme points is overwhelming. This could be a challenging and interesting research topic.

The pricing of energy plays a crucial role in applying the robust model in the real market. As the robust SCED solution is available in the proposed framework, it is possible to determine the energy price considering uncertainties. The related works are available online [25].

In this paper, the units are re-dispatched to accommodate the uncertainties. It is thus possible to introduce the flexible resource biddings in the proposed model. In this case, the recourse cost requirement becomes a useful criteria of optimizing the flexible resources in terms of the energy cost.

APPENDIX A
PROOF OF THEOREM 1

From (SP-BI), the $f(\epsilon)$ can be explained as the maximum value of $-\boldsymbol{\lambda}^\top\tilde{\boldsymbol{g}} - \boldsymbol{\lambda}^\top\boldsymbol{E}\boldsymbol{\epsilon} - \boldsymbol{\mu}^\top\tilde{\boldsymbol{\Delta}}$, which is an affine function regarding $\boldsymbol{\epsilon}$, over infinite points $(\boldsymbol{\lambda}, \boldsymbol{\mu}, \gamma)$ satisfying constraints (22-24). According to the convex theory [26], pointwise supremum over an infinite set of convex function is convex.

APPENDIX B
EXTREME POINT FORMULATION

Consider a typical budget formulation $\{\underline{u}_{it} \leq \epsilon_{it} \leq \bar{u}_{it}, L_t \leq \sum_i \epsilon_{it} \leq U_t\}$ where ϵ_{it} is the uncertainty at bus i at time t. The closed form of extreme point is

$$\begin{aligned}
\epsilon_{it} =& \underline{u}_{it} + z^u_{it}(\bar{u}_{it} - \underline{u}_{it}) \\
&+ z^L_{it}\left(L_t - \sum_k \left(\underline{u}_{kt} + z^u_{kt}(\bar{u}_{kt} - \underline{u}_{kt})\right)\right) \\
&+ z^U_{it}\left(U_t - \sum_k \left(\underline{u}_{kt} + z^u_{kt}(\bar{u}_{kt} - \underline{u}_{kt})\right)\right) \\
=& \underline{u}_{it} + z^u_{it}(\bar{u}_{it} - \underline{u}_{it}) \\
&+ z^L_{it}\left(L_t - \sum_k \underline{u}_{kt}\right) + z^U_{it}\left(U_t - \sum_k \underline{u}_{kt}\right) \\
&- (z^L_{it} + z^U_{it})\sum_k z^u_{kt}(\bar{u}_{kt} - \underline{u}_{kt}),
\end{aligned}$$

where binary variables z^u_{it}, z^L_{it}, and z^U_{it} are indicators of ϵ_{it} being bounded by $\bar{u}_{it}$, L_t, and U_t respectively. And k is an alias of i. Constraints for binary indicators are $z^u_{it} + z^U_{it} + z^L_{it} \leq 1, \sum_i z^U_{it} + z^L_{it} \leq 1, L_t \leq \sum_i \epsilon_{it} \leq U_t, \forall i, t$.

Consider the budget set shown in [8],

$$\begin{aligned}
\mathcal{U} := \{(\boldsymbol{\epsilon_1}, \cdots, \boldsymbol{\epsilon_T}) \in \mathbb{R}^{N_d} \times \cdots \times \mathbb{R}^{N_d} : \\
-\bar{\boldsymbol{u}}_{\boldsymbol{t}} \leq \boldsymbol{\epsilon_t} \leq \bar{\boldsymbol{u}}_{\boldsymbol{t}}, \forall t \quad (33) \\
\sum_m \frac{|\epsilon_{m,t}|}{u_{m,t}} \leq \Lambda^\Delta_t, \forall t\} \quad (34)
\end{aligned}$$

where $u_{m,t}$ is the bound of the uncertainty, which is an entry in vector $\bar{\boldsymbol{u}}_{\boldsymbol{t}}$. When Λ^Δ_t is an integer, as shown in [10], its extreme point can be formulated as

$$\begin{cases}
\epsilon_{m,t} = z^u_{m,t}u_{m,t} - z^l_{m,t}u_{m,t} & (35) \\
z^l_{m,t} + z^u_{m,t} \leq 1, \forall m, t & (36) \\
\sum_m (z^l_{m,t} + z^u_{m,t}) \leq \Lambda^\Delta_t, \forall t & (37) \\
z^l_{m,t}, z^u_{m,t} \in \{0, 1\}. & (38)
\end{cases}$$

APPENDIX C
PROOF OF PROPOSITION 2 AND THEOREM 2

The optimal points $[\boldsymbol{\epsilon}_1^{*\top}, \cdots, \boldsymbol{\epsilon}_T^{*\top}]^\top$ obtained by solving (SP-1-t) and (SP-2-t) are extreme points of $\mathcal{U}$. Since number of extreme points of $\mathcal{U}$ is finite, the iteration terminates finitely. On the other hand, $\mathcal{W}$ is a subset of $\mathcal{U}$, therefore solution to (MP) is a lower bound of the optimal value to (P2). Once $(\boldsymbol{x}^*, \boldsymbol{p}^*)$ from (MP) satisfies the feasible conditions in Proposition 1, the optimal value to (MP) is also the optimal value to (P2).

REFERENCES

[1] X. Guan, P. B. Luh, H. Yan, and J. A. Amalfi, “An optimization-based method for unit commitment,” *International Journal of Electrical Power & Energy Systems*, vol. 14, no. 1, pp. 9–17, 1992.
[2] S. Wang, S. Shahidehpour, D. Kirschen, S. Mokhtari, and G. Irisarri, “Short-term generation scheduling with transmission and environmental constraints using an augmented lagrangian relaxation,” *IEEE Trans. Power Systems*, vol. 10, no. 3, pp. 1294–1301, Aug. 1995.
[3] M. Shahidehpour, H. Yamin, and Z. Li, *Market operations in electric power systems*. Wiley Online Library, 2002.
[4] H. Wu, X. Guan, Q. Zhai, and H. Ye, “A systematic method for constructing feasible solution to SCUC problem with analytical feasibility conditions,” *IEEE Trans. Power Systems*, vol. 27, no. 1, pp. 526–534, 2012.
[5] S. Takriti, J. R. Birge, and E. Long, “A stochastic model for the unit commitment problem,” *IEEE Trans. Power Systems*, vol. 11, no. 3, pp. 1497–1508, 1996.
[6] L. Wu, M. Shahidehpour, and T. Li, “Stochastic security-constrained unit commitment,” *IEEE Trans. Power Systems*, vol. 22, no. 2, pp. 800–811, 2007.
[7] L. Wu, M. Shahidehpour, and Z. Li, “Comparison of scenario-based and interval optimization approaches to stochastic SCUC,” *IEEE Trans. Power Systems*, vol. 27, no. 2, pp. 913–921, 2012.
[8] D. Bertsimas, E. Litvinov, X. Sun, J. Zhao, and T. Zheng, “Adaptive robust optimization for the security constrained unit commitment problem,” *IEEE Trans. Power Systems*, vol. 28, no. 1, pp. 52–63, 2013.
[9] R. Jiang, M. Zhang, G. Li, and Y. Guan, “Two-stage network constrained robust unit commitment problem,” *European Journal of Operational Research*, vol. 234, no. 3, pp. 751 – 762, 2014.
[10] R. Jiang, J. Wang, and Y. Guan, “Robust unit commitment with wind power and pumped storage hydro,” *IEEE Trans. Power Systems*, vol. 27, no. 2, pp. 800 – 810, 2012.
[11] B. Zeng and L. Zhao, “Solving two-stage robust optimization problems using a column-and-constraint generation method,” *Operations Research Letters*, vol. 41, no. 5, pp. 457–461.
[12] C. Zhao and Y. Guan, “Unified stochastic and robust unit commitment,” *IEEE Trans. Power Systems*, vol. 28, no. 3, pp. 3353–3361, 2013.
[13] J. Warrington, P. Goulart, S. Mariethoz, and M. Morari, “Policy-based reserves for power systems,” *IEEE Trans. Power Systems*, vol. 28, no. 4, pp. 4427–4437, Nov 2013.
[14] R. Jabr, “Adjustable robust OPF with renewable energy sources,” *IEEE Trans. Power Systems*, vol. 28, no. 4, pp. 4742–4751, Nov 2013.
[15] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, “Adjustable robust solutions of uncertain linear programs,” *Math. Programming*, vol. 99, no. 2, pp. 351–376, 2004-03-01.
[16] J. Warrington, C. Hohl, P. Goulart, and M. Morari, “Rolling Unit Commitment and Dispatch With Multi-Stage Recourse Policies for Heterogeneous Devices,” *IEEE Transactions on Power Systems*, 2015, early Access.
[17] A. Lorca, A. Sun, E. Litvinov, and T. Zheng, “Multistage adaptive robust optimization for the unit commitment problem,” 2014.
[18] M. Ilic, L. Xie, and J.-Y. Joo, “Efficient Coordination of Wind Power and Price-Responsive Demand-Part I: Theoretical Foundations,” *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 1875–1884, 2011.
[19] A. Parisio and L. Glielmo, “Stochastic model predictive control for economic/environmental operation management of microgrids,” in *Europen Control Conference*, July 2013, pp. 2014–2019.
[20] H. Ye and Z. Li, “Robust security-constrained unit commitment with recourse cost requirement,” in *Power Energy Society General Meeting, 2015 IEEE*, July 2015, pp. 1–5.
[21] J. Wang, M. Shahidehpour, and Z. Li, “Security-constrained unit commitment with volatile wind power generation,” *IEEE Trans. Power Systems*, vol. 23, no. 3, pp. 1319–1327, 2008.
[22] Z. Li and M. Shahidehpour, “Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets,” *IEEE Trans. Power Sys.*, vol. 20, no. 2, pp. 1079–1088, May 2005.
[23] D. J. White, “A linear programming approach to solving bilinear programmes,” *Math. Programming*, vol. 56, no. 1-3, pp. 45–50, 1992.
[24] I. Gurobi Optimization, *Gurobi Optimizer Reference Manual*, 2014. [Online]. Available: http://www.gurobi.com
[25] H. Ye, Y. Ge, M. Shahidehpour, and Z. Li, “Market clearing for uncertainty, generation reserve, and transmission reserve—Part I: Theory,” arXiv:1507.01540 [math.OC], http://arxiv.org/abs/1507.01540.
[26] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2009.

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